Model-Order Reduction Aided Harmonic Balance Analysis

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Abstract
This paper presents an approach to speed up the harmonic balance analysis and reduce the memory consumption by utilizing model-order reduction. A simulation example is presented to verify the efficiency of the method.

1 Introduction
Modern RF and microwave circuits are typically large, consisting of thousands of components and nodes. During the design process, there is a need to perform computationally demanding numerical simulations to verify the functionality of the circuit.

The harmonic balance (HB) method [1, 2] is a frequency-domain analysis technique to solve the periodic and quasi-periodic steady state. It is widely used for RF and microwave circuit analysis.

Since the HB equations easily become huge, the memory consumption and simulation time becomes a bottle neck for efficient circuit design. Thus, model-order reduction (MOR) [3, 4] is a handy approach where the idea is to simplify complex circuit models by approximating the model by simpler one. In the best case, the reduced-order model (ROM) is smaller size compared to the original, but still describes the original system perfectly. In practice, the reduction process generates some error in the ROM in a trade-off for a smaller system. There are MOR methods for linear and nonlinear circuits, but in this paper, only linear MOR for the linear parts of a circuit is considered.

This paper present a model-order reduction approach to improve the efficiency of the HB analysis by reducing the linear part of the circuit.

2 Background
2.1 Harmonic Balance
In HB analysis, the variables are presented in terms of Fourier coefficients:

\[ x(t) = \sum_{k=-N}^{N} X_k e^{j\omega_k t}, \]  

(1)

where \( N \) is the number of harmonic frequencies, \( X_k \) is the \( k \)th Fourier coefficient, and \( \omega_k \) the \( k \)th harmonic frequency. The total number of unknowns is \((2 \times N + 1) \times n\), where \( n \) is the number of the nodes of a circuit. Since the HB equation system easily becomes huge, they are usually solved using the inexact Newton [5] method, i.e. the Newton–Raphson method with an iterative linear solver like GMRES [6].

The frequency-domain HB equations are

\[ F(X) = I(X) + j\Omega Q(X) + U, \]  

(2)
where \( \mathbf{U} \) is the vector of excitation currents, \( \mathbf{X} = \Gamma \mathbf{x}(t) \) is the nodal voltage vector, and \( \mathbf{I} = \Gamma \mathbf{i}(t) \) and \( \mathbf{Q} = \Gamma \mathbf{q}(t) \) are the nonlinear nodal current and charge vectors, respectively. \( \Gamma \) is the operator meaning (multidimensional) discrete Fourier transform (DFT), and \( \Omega \) is the frequency-domain derivative matrix.

In practice, the DFT is usually replaced with the fast Fourier transform (FFT), which is the efficient implementation of the DFT.

The whole HB analysis (using inexact Newton method) is as follows:

1. Set initial guess \( \mathbf{X}_0, k = 0 \).
2. Inverse discrete Fourier transform (IDFT): \( \mathbf{x}(t) = \Gamma^{-1} \mathbf{X}_k \).
3. Compute nonlinear functions \( \mathbf{i}(\mathbf{x}(t)) \) and \( \mathbf{q}(\mathbf{x}(t)) \).
4. DFT: \( \mathbf{I} = \Gamma \mathbf{i} \) and \( \mathbf{Q} = \Gamma \mathbf{q} \).
5. Solve new iterate for \( \mathbf{X}_{k+1} \).
6. Check convergence. If no convergence go to step 2.
7. Solution found.

### 2.2 Model-Order Reduction

There are many MOR methods presented in the literature, but a typical approach in MOR is to reduce the linear circuit such that the input-output behaviour, i.e. \( \gamma \) parameters, remains approximately the same. This is done by reducing the number of internal nodes and components.

The MNA equations of an linear \( N \)-port can be expressed as follows:

\[
\begin{aligned}
\mathbf{C} \frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} &= -\mathbf{G}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \\
\mathbf{i}_p(t) &= \mathbf{L}^T\mathbf{x}(t),
\end{aligned}
\]  

(3)

where \( \mathbf{x}(t) \) contains the node voltages and branch currents of inductances and voltage sources, \( \mathbf{u} \) and \( \mathbf{i}_p \) denote the port voltages and currents. \( \mathbf{B} = \mathbf{L} \) where \( \mathbf{B} \) is a selector matrix consisting of ones, minus ones and zeroes. \( n \) is the total number of unknowns. The \( \gamma \) parameters are defined as \( \mathbf{A} \equiv -\mathbf{G}^{-1}\mathbf{C} \) and \( \mathbf{R} \equiv \mathbf{G}^{-1}\mathbf{B} \). Taking the Laplace transformation of (3) and solving for the port current variables, the \( \gamma \)-parameter matrix \( \mathbf{Y}(s) \) is

\[
\mathbf{Y}(s) = \mathbf{L}^T(1-s\mathbf{A})^{-1}\mathbf{R}.
\]  

(4)

PartMOR [7] is a recently proposed MOR method that is a partitioning-based RLC-in–RLC-out MOR method that generates passive reduced-order circuits with positive-valued RLC elements (see Fig. 1). The method first divides the original circuit into small partitions, which can then be approximated with low-order RLC macromodels. The approximation is performed by generating the \( \gamma \)-parameter moment series at DC and infinity,

\[
\mathbf{Y}(s) = \mathbf{M}_0 + \mathbf{M}_1s + \mathbf{M}_2s^2 + \cdots,
\]  

(5)

\[
\mathbf{Y}(s) = \mathbf{N}_0 + \mathbf{N}_1\frac{1}{s} + \mathbf{N}_2\frac{1}{s^2} + \cdots,
\]  

(6)

and matching the first few moments from both of the series with one of the presented RLC macromodels. Since the method matches moments both at DC and infinity, a good approximation can be achieved over a wide frequency band.

Note that now the original hierarchy is preserved. All the sources and nonlinear elements are untouched, and only the linear parts are reduced.
3 Numerical simulation results

In order to show that speeding up the HB analysis is possible by using the MOR approach to speed up, a numerical simulation example is presented. The simulation example is a circuit consisting of two MOSFET inverters with a large \( RLCM \) interconnect between them. The simulation is a one-tone HB analysis with 5 harmonics performed using APLAC [8]. The reduction is done by using PartMOR with its extensions presented in [9, 10].

Table 1 presents the simulation results. Here, \( n, n_x, R, C, L, \) and \( M \) denotes the number of nodes, unknowns, resistances, capacitances, inductances, and mutual inductances, respectively. The simulation time \( t \) and memory consumption (Mem) are also presented. The Fig. 2 shows that the output spectra of the original circuit and the reduced circuit are approximately the same while the speed-up is enormous.

4 Conclusion

The MOR-aided HB analysis was presented. The simulation example showed that the PartMOR method was capable of reducing the linear part of the circuit such that the overall performance of the HB analysis was improved.

5 References

References


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Table 1: Simulation example.

<table>
<thead>
<tr>
<th></th>
<th>( n )</th>
<th>( n_x )</th>
<th>R</th>
<th>C</th>
<th>L</th>
<th>M</th>
<th>( t/s )</th>
<th>Mem/Mb</th>
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<td>66022</td>
<td>2000</td>
<td>3538</td>
<td>1998</td>
<td>1538</td>
<td>165.0</td>
<td>2477</td>
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<tr>
<td>reduced</td>
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<td>2563</td>
<td>154</td>
<td>2775</td>
<td>75</td>
<td>669</td>
<td>0.6</td>
<td>62</td>
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</tbody>
</table>
Fig. 2: Spectrum of the original circuit output voltage (-X-) and the the reduced circuit output voltage (-O-). The spectra are distinguishable.


