Achieving High Absorbing Efficiency for an Electromagnetically Asymmetric Slab Made of Hyperbolic Metamaterials

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Abstract

An anisotropic medium with hyperbolic dispersion relation and tilted optical axis has been found to support two asymmetric propagating waves within its volume. With careful selection of design parameters one can achieve perfect matching with free space and huge spatial oscillation for the transmitted wave. If moderate losses are added, these properties can be used for the construction of very thin and efficient electromagnetic absorbers.

1 Introductory Comment

Devices that absorb the electromagnetic radiation are extensively applicable to real-world configurations functional in a wide range of oscillation frequencies. One of the first microwave absorbers, consisted of a matched lossy sheet placed over a metal plate at a quarter wavelength distance, has been patented by Salisbury in 1952 [1]. Modern materials such as graphene have been employed recently to construct attenuators and absorbers which achieve very small levels of reflection in the optical range [2, 3]. Similarly, vertically aligned single-walled carbon nanotubes have been utilized to experimentally demonstrate high absorbing performances [4].

Application of metamaterials makes possible the design of resonant elements combining electric and magnetic properties [5]. There are three beneficial characteristics when one utilizes metamaterials: (i) frequency scaling due to geometrical scalability, (ii) independence of electric and magnetic resonances and (iii) matching with free space through effective permittivity and permeability. Perfect absorption in metamaterials has been reported in a variety of frequency bands such as mid-infrared [6]. Particularly interesting for these operations, is a class of metamaterials which possesses a hyperbolic-type dispersion relation. Some general wave properties of hyperbolic (or indefinite as they are also called) media with wedge-like shapes are discussed in [7].

In the present paper, we propose the concept of an asymmetric hyperbolic media and examine its capacity in light absorbing. Asymmetry appears as a difference in normal wave vector components for waves, propagating downward and upward with respect to the horizontal interfaces of the slab. These properties are exhibited in uniaxial crystal slabs with optical axes tilted with respect to the interfaces. This phenomenon takes place in any uniaxial media; however, new effects potentially prospective for creation of a new type of optical absorbers are appeared in the case of hyperbolic media.

2 Mathematical Formulation

The analyzed two-dimensional (2D) configuration is shown in Fig. 1, where the two alternative Cartesian coordinate systems: the main unprimed, \((x, y, z)\) and primed auxiliary \((x', y', z')\) are also defined. A slab of thickness \(W\) is filled with an anisotropic substance whose relative permittivity tensor in the primed coordinate system is expressed as follows:

\[
[e_r'] = \begin{bmatrix} \varepsilon_\perp & 0 & 0 \\ 0 & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{bmatrix},
\]  
(1)
with $\varepsilon_\perp > 0$ and $\varepsilon_\parallel < 0$. In other words, the material in the primed coordinate system is a hyperbolic medium, namely a medium with hyperbolically-shaped isofrequency curves (wavenumber solutions verifying the dispersion equation). The corresponding relative permittivity tensor of the material with respect to the unprimed coordinate system $[\varepsilon_r]$ is readily found by proper multiplication with the well-known rotation matrices by the tilt angle $\xi$ (around $y = y'$ axis). The structure is excited by a TM plane wave propagating along the direction that forms an angle $\theta$ with the negative $z$ semi-axis. Obviously, the transverse wave impedance of the incident field is computed as: $Z_{inc} = \eta \cos \theta$, where $\eta = \sqrt{\mu_0/\varepsilon_0} = 120\pi \Omega$ is the free-space wave impedance ($\varepsilon_0$, $\mu_0$ are the constituent electromagnetic parameters of vacuum). A harmonic time dependence $e^{-i\omega t}$ is adopted and suppressed throughout the analysis (with operating wavelength $\lambda = 2\pi c/\omega$, $c$ is the speed of light).

Fig. 1: The physical configuration of the absorbing slab. It is filled with a tilted hyperbolic medium with losses, while the structure is excited by an obliquely incident TM plane wave.

After obtaining the general solution of the vectorial Helmholtz equation and imposing the necessary boundary conditions, the supported normal wavenumbers by the slab are rigorously determined as follows:

$$k^{(1,2)}_z = \frac{k_x e_{xz} \pm \sqrt{(e_{xx}^2 - e_{xx} e_{zz})(k_x^2 - k_0^2 e_{zz})}}{e_{zz}},$$

where $k_x = k_0 \sin \theta$ is the transverse wavenumber imposed by the excitation and the required phase matching. The parameters $\{e_{xx}, e_{zz}, e_{xz}\}$ are the corresponding elements of the aforementioned matrix $[e_r]$. The symbol $k_0 = \omega/c$ is reserved for the free-space wavenumber. Note finally that the wave with $k_z = k_z^{(1)}$ is the one propagating upwards, namely along the negative $z$ semi axis ($-z$). Similarly, the wave with $k_z = k_z^{(2)}$ is traveling downwards ($+z$) or upwards ($-z$) in proportion to the oscillation wavelength $\lambda$.

Let us focus on a special case where certain beneficial characteristics are observed. Assume that the following conditions are (at least approximately) fulfilled:

$$\varepsilon_\perp \approx 1, \quad \varepsilon_\parallel \approx -1 + i\delta, \quad \theta \approx -\xi \approx 45^\circ,$$

with $1 > \delta > 0$. Under the conditions stated above, the diagonal elements of the tensor $[e_r]$ vanish and the off-diagonal ones tend to $-1 + i\delta$. Accordingly, the difference between the wavenumbers of the two supported asymmetric waves goes to infinity. In particular:

$$k_z^{(1)} \to -k_0 \cos \theta, \quad \text{Im}(k_z^{(2)}) \to +\infty,$$

with $0 < \delta \to 0$. Such a result of unboundedly increasing imaginary part of wavenumber is very important since corresponds to a spatially rapid absorption of the electromagnetic power even if
the parameter of losses is small. To put it alternatively, if one adds some moderate losses \( \delta \) in the medium constituting the slab, one would obtain a very large \( |\Im[k_z^{(2)}]|W \) (even if \( W \) is small compared to \( \lambda \)). Therefore, the dumping factor \( e^{-\Im[k_z^{(2)}]W} \) would vanish the transmitted field into the slab by converting its carried power into thermal form (absorption by dielectric losses).

What is additionally intriguing is that under the aforementioned regime \( \Im[k_z^{(2)}] \rightarrow +\infty \), the two asymmetric waves supported by the anisotropic medium do not experience reflection from slab interfaces. More specifically, it has been found that the corresponding wave impedances (for the waves (1) and (2), respectively) take the (common) form:

\[
Z_{1,2} = \left. \frac{E_x}{H_y} \right|_{z=0} = \mp \eta \cos \theta = \mp Z_{\text{inc}},
\]

where \( \theta = 45^\circ \). In this way, we have achieved: (i) negligible back reflections due to impedance matching \( Z = Z_{\text{inc}} \), (ii) substantial wave damping and attenuation with small thickness \( W/\lambda \) and small losses due to the huge \( \Im[k_z^{(2)}] \). These two features constitute a highly effective electromagnetic absorber.

Fig. 2: (a) The relative longitudinal wavenumbers \( k_z/k_0 \) as function of the operating wavelength \( \lambda \) for several tilt angles \( \xi \). (b) The absorption coefficient \( A \) as function of the operating wavelength \( \lambda \) for several doped silicon concentrations \( N \).

### 3 Graphs and Discussion

In Fig. 2(a), we consider a composite of tilted doped silicon nanowires as the indefinite hyperbolic medium of the slab. With the concentration of carriers in doped silicon \( N = 5 \times 10^{21} \text{ cm}^{-3} \) and the volume density of Si nanowires in the composite \( p = 0.1 \), the conditions (3) are approximately satisfied at the wavelength \( \lambda_0 \approx 1.37 \mu \text{m} \), where \( \Re(\varepsilon_\perp) \approx 1.25 \), \( \Im(\varepsilon_\perp) \approx 0.0025 \), \( \Re(\varepsilon_\parallel) = -1 \), \( \Im(\varepsilon_\parallel) \approx 0.1 \). One can see that the imaginary part of \( k_z^{(2)} \) becomes very large in the vicinity of \( \lambda_0 \). So, even low material losses would certainly cause high wave attenuation at a short propagation distance under the perfect matching with free space.

In Fig. 2(b), we show the dependence of the absorption coefficient \( A \) (the ratio of the absorbed power and the incidence power), calculated for different concentrations of carriers. It is remarkable, that the thickness resonance is absent due to the difference in the propagation constants for upward and downward waves within the slab. Furthermore, the best absorption is observed for the tilt angle \( \xi = -40^\circ \), despite that the ideal absorption condition (for \( \theta = 45^\circ \)) corresponds to \( \xi = 45^\circ \). Such a difference is clearly attributed to the deviation of material parameters from the perfect absorption conditions, particularly, because \( \varepsilon_\perp \approx 1.25 \).
In Fig. 3, we demonstrate the results of ANSOFT HFSS numerical simulations for a Gaussian beam incident onto a tilted absorbing structure. The optical axis is parallel to the $z'$-axis, as in Fig. 1. The absorbing effect is almost perfect and only minor reflections occur.

Fig. 3: Field density plot obtained by commercial simulation software ANSOFT HFSS. Absorption of the Gaussian beam, shown in the $x'z'$ plane (left) and $x'y'$ plane (right).

4 Concluding Remarks

In this work, we have shown that hyperbolic media with tilted optical axes exhibit unusual properties and allow realization of materials, supporting propagation of waves, characterized with very large wave vector components under perfect matching regime with free space. Simultaneous fulfillment of these conditions cannot be achieved in any known material and opens a door for creation of a new class of optically ultra thin absorbers.

References


