Measurements of Effective Material Parameters of Periodic and
Random Composite Materials

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Abstract

In the literature on electromagnetic composite materials with exotic or unusual properties (so-called metamaterials), it is common to find the homogenized material parameters of a realized or simulated material sample by using the well-known Nicolson-Ross-Weir material parameter extraction method. This method relates the scattering parameters of a finite-thickness material slab to the permittivity and permeability of the effective homogenized material comprising the slab. Effective parameters of many metamaterials studied in the literature exhibit a so-called anti-resonance, which is a nonphysical feature, in at least one of the parameters that are retrieved this way. Many previous works have either claimed or assumed that the anti-resonance is an effect caused by the periodicity of the composite material. Here we investigate the question if the sample periodicity is indeed essential for appearance of anti-resonances in extracted material parameters.

1 Introduction

The emergence of the so-called anti-resonance in the experimentally or numerically retrieved material parameters (permittivity \(\varepsilon\), permeability \(\mu\)) of many composite materials (e.g. [1–4]) has recently aroused many discussions on the physical meaning of such parameters, see e.g. [5–9]. The anti-resonant response means that in the low-loss parts of the frequency spectrum the effective material parameter decays with increasing frequency (non-Foster behavior) and in the absorptions bands the imaginary part of the effective material parameter has the sign which corresponds to power gain instead of loss. These features indicate violation of causality and passivity requirements in this effective materials model [7–9]. Here we are interested in the question of relation of the appearance of parasitic anti-resonances to the periodicity or randomness of the sample microstructure. To the best of our knowledge, earlier studies of the anti-resonance in retrieved parameters and its interpretations considered periodical arrays of inclusions, either 3D infinite lattices or 2D-periodical single- or multiple-layer composite slabs. While it appears to be well establishes that the nature of the nonphysical anti-resonance peaks is in spatial dispersion due to electrically finite sizes of resonant unit cells, the role of the periodicity of the lattice remains unclear. However, often this nonphysical behavior of the material parameter(s) is explained by the periodicity of the composite material, e.g., [5, 6].

2 Parameter extraction method and experimental sample

As in most other works, we also use the classical retrieval procedure, the Nicolson-Ross-Weir (NRW) method, which relates the complex reflection and transmission coefficients of a composite slab to the permittivity and permeability of the effective homogeneous material filling the slab volume. It is necessary to note that in the NRW method inaccurate results and parasitic anti-resonant peaks can appear also close to the thickness (Fabry-Perot) resonances of the sample. However, the problem of the thickness resonances can be overcome in a number of ways (e.g., measuring samples of different thicknesses), and several robust techniques for material parameter extraction exist. In this work we use the extraction algorithm of paper [10].
The composite material sample used in this work is designed to exhibit a resonant effective permittivity. This is accomplished by a material comprising small electric dipoles embedded in a dielectric medium. The electric dipoles are simple copper strips having a geometry that makes the dipoles electrically small at their resonant frequency [11] (see Fig. 1).

The aperiodycity of the composite material is twofold: aperiodicity in the transversal plane and aperiodycity in the longitudinal direction. The transversal plane aperiodicity is realized by randomly positioning the inclusions on the FR4 boards. Four different FR4 boards with a randomized distribution of the inclusions were manufactured. The aperiodicity in the longitudinal direction is controlled via the teflon thickness between adjacent FR4 boards. To randomize the positions of these boards along the z-axis, we have computed four sets of random values for \( \Delta z \), with normal distribution and standard deviation of 0.5 mm.

In order to measure the reflection and transmission coefficients of periodic and aperiodic composite material slabs, we have used a bistatic measurement setup at the Microwaves and Radar Institute of the German Aerospace Center (DLR). The reflection and transmission coefficients of the measured composite materials are also modelled numerically with Ansys HFSS software. Due to the huge computational cost of simulating the whole slab with random arrangement of particles, we are forced to numerically simulate the slabs as transversally infinite (employing periodic boundary conditions), thus making it impossible to introduce the aperiodicity in the transversal (xy) plane. However, there is no problem in the numerical analysis of the material slab that is periodic, nor the ones that are aperiodic only in the longitudinal (z) direction.

3 Results

As the reference sample we study a periodic composite material slab comprising four periodic FR4 boards. The magnitudes of the measured and simulated scattering parameters (S-parameters) are shown in Fig. 2, together with the extracted refractive index \( (n) \), the relative permittivity \( (\varepsilon_r) \), and the relative permeability \( (\mu_r) \). From Fig. 2 we can conclude that the numerical and experimental results are in good agreement with each other, but show a slight shift in the resonant frequency.
Most importantly, the permittivity is resonant whereas the permeability is anti-resonant, with the imaginary part of the permeability having the “wrong”, i.e., nonphysical, sign in the frequency range of the anti-resonance (with the chosen time dependence any passive material should have the imaginary parts of both permittivity and permeability negative).

By changing the number of the thin teflon sheets between the adjacent FR4 boards, we can control the period of our composite material slab in the longitudinal direction. In the numerical and experimental study we employ four randomized sets of samples, and all four random samples behave very similarly near the resonance and anti-resonance of $\varepsilon$ and $\mu$. The numerical as well as experimental results are given in Fig. 3, in the same convention as for the reference periodic sample. Again, the scattering data is omitted in a small frequency band, corresponding to the same stopband as in the periodic case.

Let us concentrate only on the material parameter resonance and anti-resonance that occur at around 7 GHz. Similarly to the periodic sample, the experiment results in the resonance and anti-resonance at slightly higher frequencies than in the case of the numerical model, but overall the numerical and experimental results are in good agreement with each other. The most interesting issue is that we clearly see that the anti-resonance of $\mu$ remains even in the aperiodic samples.

In the case of all four aperiodic samples, the permittivity is resonant whereas the permeability is anti-resonant. In the presentation we will show and discuss also the samples where the positions of inclusions on each substrate has been randomized. The experimental results exhibit anti-resonant peaks also for these fully random samples.
4 Conclusions

In this work we have experimentally shown that the anti-resonance artefact is not a direct consequence of the periodicity of the material, since all the studied material samples, both periodic and aperiodic ones, clearly show a similar frequency response when it comes to the resonance and anti-resonance of the extracted material parameters. Thus, we have proven that the physical reason of the anti-resonant behaviour of effective parameters is the resonant response of finite-size inclusions and unit cells, while the particular arrangement of these cells is not essential. Our results support the theory which treats the antiresonance as a deficiency of the retrieval procedure neglecting the surface effects [9, 12].

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References


