An Efficient Close to Optimal Radio Resource Allocation Mechanism towards LTE Downlink Transmission

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INTRODUCTION

Long Term Evolution (LTE) is the well known radio access technology specified by one of the global standards developing organization, the Third Generation Partnership Project (3GPP)[1]. In this paper we consider a problem of radio resource allocation (RA) which can be adapted towards RA for LTE downlink transmission. Since the LTE downlink transmission scheme is based on Orthogonal Frequency Division Multiple Access (OFDMA), we abstract the main problem in to a radio RA problem in an OFDMA system. OFDMA plays a major role in the physical layer specifications of future wireless technologies which is not confined to LTE (e.g., WIMAX, IMT-A) [2]. In general OFDMA based transmission systems, the transmit power and sub-carriers are dynamically assigned to the users, based on their channel state information (CSI) to optimize a certain performance. Indeed this process requires solving combinatorial optimization problems.

The main contribution of this paper is to provide a low complexity very close to optimal resource allocation algorithm for joint optimization of multiuser subcarrier assignment and power allocation which is a simple extension of our previous work [3]. Numerical results are provided to compare the performance of the proposed algorithm to Lagrange relaxation based suboptimal methods [4] as well as to optimal exhaustive search based method. We did not consider each and every fine details of the LTE time/frequency domain structures of the frames which is out of the scope of the main concern. But the proposed solution method indeed can be used as a basic building block for more general resource allocation algorithms for practical LTE downlink transmissions after having appropriate modifications.

SYSTEM MODEL AND PROBLEM FORMULATION

Consider a single antenna OFDMA downlink transmission with K users and M subcarriers. The signal received by user k in subcarrier m can be expressed as

$$r_{km} = h_{km} x_{km} \sqrt{p_{km}} + w_{km}, \quad k = 1, \ldots, K, \ m \in \mathcal{S}_k, \tag{1}$$

where k is the user index, m is the subcarrier index, $\mathcal{S}_k$ denotes the set of subcarriers allocated to user k, x_{km} is the transmitted signal, p_{km} is the power allocated, h_{km} is channel frequency response and w_{km} is the received noise. We assume that h_{km} is time-invariant and its value is available at the base station. The noise samples are assumed to be independent and identically distributed as $w_{km} \sim \mathcal{CN}(0, \sigma_{km}^2)$. We denote by $c_{km} = \frac{|h_{km}|^2}{\sigma_{km}^2}$ the channel signal-to-noise ratio (SNR) of kth user in subcarrier m and by β_k the weight associated with the rate of user k. The weighted sum-rate maximization problem subject to a sum-power constraint P_T can be formulated as [3]

$$\begin{array}{ll} \text{maximize} & \sum_{k=1}^{K} \sum_{m \in \mathcal{S}_k} \beta_k \log_2(1 + p_{km} c_{km}) \\ \text{subject to} & \sum_{k=1}^{K} \sum_{m \in \mathcal{S}_k} p_{km} = P_T \\ & \mathcal{S}_k \cap \mathcal{S}_l = \emptyset, \quad \forall k \neq l \\ & p_{km} \geq 0, \quad k = 1, \ldots, K, \ m = 1, \ldots, M, \end{array} \tag{2}$$

where variables are p_{km} and $\mathcal{S}_k$.

APPROXIMATED PRIMAL DECOMPOSITION BASED ALGORITHM

By considering an equivalent virtual system and introducing M new variables $\bar{p}_m = \sum_{k=1}^{K} p_{km}$, $m = 1, \ldots, M$, problem (2) can be be decomposed into a master problem and M subproblems, one subproblem for each subcarrier $m = 1, \ldots, M$ [3]. For a given subcarrier m, the subproblem is given by

$$\begin{array}{ll} \text{maximize} & \sum_{k=1}^{K} \beta_k \log_2 \left(1 + \dfrac{p_{km}}{\bar{p}_m - p_{km} + c_{km}^{-1}} \right) \\ \text{subject to} & \sum_{k=1}^{K} p_{km} = \bar{p}_m \\ & p_{km} \geq 0, \quad k = 1, \ldots, K, \end{array} \tag{3}$$

where variables are p_{km}, $k = 1, \ldots, K$. The master problem can be expressed as

$$\begin{array}{ll} \text{maximize} & \sum_{m=1}^{M} f_m^\star(\bar{p}_m) \\ \text{subject to} & \sum_{m=1}^{M} \bar{p}_m = P_T \\ & \bar{p}_m \geq 0, \; m = 1, \ldots, M, \end{array} \tag{4}$$

where variables are $\bar{p}_m$ and $f_m^\star(\bar{p}_m)$ represents the optimal value of subproblem (3) for fixed $\bar{p}_m$. Let us denote by $\mathcal{P}$ the feasible set of problem (3). Even though the subproblem (3) is not a convex optimization problem, from [5, Corollary 32.3.4] it follows that the solutions of subproblems (3) must be achieved at one of the vertices of the polyhedral set $\mathcal{P}$. Consequently, the solutions of the M subproblems can be expressed as

$$[p_{1m}^\star, \ldots, p_{Km}^\star] = \bar{p}_m \mathbf{e}_{j_m}^{\mathrm{T}}, \qquad j_m = \arg\max_k \, (1 + \bar{p}_m c_{km})^{\beta_k}, \quad m = 1, \ldots, M. \tag{5}$$

where j_m represents the index of the user allocated to mth subcarrier. By substituting (5) in the objective of (3), $f_m^\star(\bar{p}_m)$ can be expressed as

$$f_m^\star(\bar{p}_m) = \beta_{j_m} \log_2 (1 + \bar{p}_m c_{j_m m}) = \max_k \beta_k \log_2 (1 + \bar{p}_m c_{km}) \, , \quad m = 1, \ldots, M. \tag{6}$$

Note that the function $f_m^\star(\bar{p}_m)$ is the pointwise maximum of a set of concave functions and thus $f_m^\star(\bar{p}_m)$ is not a concave function w.r.t $\bar{p}_m$ [6] in general. Thus, standard convex optimization tools (e.g., subgradient based methods) can not be directly applied to solve master problem (4). The trick is to use the following concave lower bound on the objective function of problem (4) so that approximated master problem is solved optimally.

$$\sum_{m=1}^{M} \beta_{j_m^{(i)}} \log_2 \left(1 + \bar{p}_m c_{j_m^{(i)} m} \right) \leq \sum_{m=1}^{M} f_m^\star(\bar{p}_m), \quad [\bar{p}_1, \ldots, \bar{p}_M]^T \in \bar{\mathcal{P}} \tag{7}$$

Thus, the proposed algorithm can be summarized as follows:

Algorithm 1 *Approximated primal decomposition for OFDMA weighted sum-rate maximization with parallel initializations*

1. *initialization: Given the number of random initialization points N_{rand}. Let $i = 1$ and $l = 0$.*
   - *(a) Uniform power init.: $\bar{p}_m^{(i)}(0) = P_T/M, \; m = 1, \ldots, M$*
   - *(b) Random power init: $\bar{p}_m^{(i)}(l) = \mathcal{U}(0,1).P_T/M, \; m = 1, \ldots, M, \; l = 1, \ldots, N_{rand}$, where $\mathcal{U}(0,1)$ is a random number which is uniformly distributed between 0 and 1.*
2. *if $l \leq N_{rand}$ go to step 3, otherwise go to step 6.*
3. *solve the M subproblems (3) for $\bar{p}_m = \bar{p}_m^{(i)}(l)$ and return $j_m^{(i)}(l)$, computed according to (5). Let $j_m^{(i)} = j_m^{(i)}(l)$ and $g = \sum_{m=1}^{M} \beta_{j_m^{(i)}} \log_2 \left(1 + \bar{p}_m c_{j_m^{(i)} m} \right)$.*

4. *solve the following approximation of master problem (4)*

$$\begin{array}{ll} \textit{maximize} & \sum_{m=1}^{M} \beta_{j_m^{(i)}} \log_2 \left(1 + \bar{p}_m c_{j_m^{(i)} m}\right) \\ \textit{subject to} & \sum_{m=1}^{M} \bar{p}_m = P_T \\ & \bar{p}_m \geq 0, \quad m = 1, \ldots, M, \end{array} \tag{8}$$

and return the solution $\bar{p}_m^\star$*; let* $\bar{p}_m^{(i+1)}(l) = \bar{p}_m^\star$ *and* $g^\star(l) = \sum_{m=1}^{M} \beta_{j_m^{(i)}} \log_2 \left(1 + \bar{p}_m^\star c_{j_m^{(i)} m}\right)$.

5. *check a stopping criteria* $g^\star(l) = g$*:*

   (a) *if the stopping criteria is not satisfied let* $i = i + 1$ *and go to step 3.*

   (b) *otherwise store the power allocation* $\bar{p}_m^{(i+1)}(l)$*, subcarrier allocation* $j_m^{(i)}(l)$*, and the associated objective value of problem (8)* $g^\star(l)$ *for* l*th iteration, let* $l = l + 1$ *and go to step 2.*

6. *Choosing the best power and subcarrier allocation:*

$$l^\star = \underset{l \in \{0 \ldots, N_{rand}\}}{\arg\max} \; f^\star(l). \tag{9}$$

choose the power allocation and the subcarrier allocation stored in Step 5(b) which are associated with $l^\star$.

The solution of problem (8) solved at step 3 is found by multilevel waterfilling (see [3] for more details).

NUMERICAL RESULTS

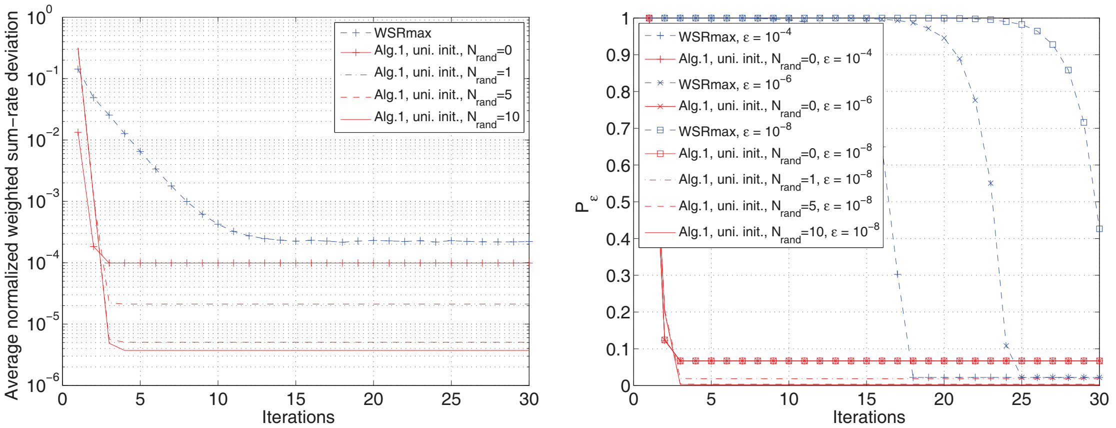
The performance of the proposed algorithm is compared to the dual decomposition based algorithm proposed in [4], denoted as WSRmax, as well as to the optimal algorithm based on exhaustive search. For a fair comparison, we first define the metric $\Delta C_{\text{weighted sum}} = \mathrm{E}\left\{\frac{|C_{\text{opt}} - \hat{C}_{\text{subopt}}|}{C_{\text{opt}}}\right\}$, the average normalized weighted sum-rate deviation (ANWSRD), where, C_{opt} is the optimal weighted sum-rate value obtained using optimal exhaustive search, $\hat{C}_{\text{subopt}}$ is the estimated objective value from either Algorithm 1 or the WSRMax algorithm and expectation $\mathrm{E}\{.\}$ is taken w.r.t channel realization. Since for large M the computational complexity of optimal exhaustive search is prohibitively expensive. Thus, in our simulations we considered a small scale OFDMA system with $M = 8$ subcarriers and $K = 2$ users. It should be emphasized that this limitation is solely for the comparison purpose of Algorithm 1 with the optimal solution and not a limitation of our Algorithm 1. A uniform power delay profile with 4 channel taps is considered. We assume $\sigma_{km}^2 = \sigma^2, k = 1, \ldots, K,\ m = 1, \ldots, M$ and define SNR per subcarrier as $\frac{P_T}{M.\sigma^2}$.

Figure 1(a) shows the convergence behavior of the considered algorithms with SNR$= 10$dB for $N_{\text{rand}} = 0, 1, 5$ and 10. The weights of the users are $[1, 2]$. The floor of the curves is due to the suboptimality of the algorithms. The results show that Algorithm 1 converges faster than the WSRMax algorithm and provides smaller values of ANWSRD. Specifically, for any value of N_{rand} Algorithm 1 requires only 3 iterations on average to achieve a constant level of ANWSRD whilst the WSRMax algorithm requires around 15 iterations to reach a constant level. Even with just uniform initialization (i.e., the case of $N_{\text{rand}} = 0$) the ANWSRD values achieved by proposed algorithm is smaller as compared to that of WSRMax algorithm. Remarkable reduction of ANWSRD is obtained when N_{rand} is changed from 0 to 1. As N_{rand} increased the incremental gains that can be obtained in terms of ANWSRD is reduced.

Then we compare the behavior of Algorithm 1 and the WSRMax algorithms using the following metric $P_\varepsilon = \text{Prob}\left\{|\ C_{\text{opt}} - \hat{C}_{\text{subopt}}\ | > \varepsilon\right\}$, the probability of missing the global optimal, where ε is a small number which quantifies the maximum admissible deviation between C_{opt} and $\hat{C}_{\text{subopt}}$. It is considered that the global optimum is missed if $\hat{C}_{\text{subopt}}$ is more than ε away from C_{opt}.

Figures 1(b) uses the same simulation setup as that in Figure 1(a) and depict the variation of probability of missing the global optimal, P_ε with the number of iterations. The floor of probability

P_ε is again due to the suboptimality of both algorithms. The influence of ε on P_ε for fixed N_{rand} (= 0) is totally indistinguishable in case of proposed algorithm. This behavior shows that the proposed algorithm can arrive very close to optimal solutions within a very small number of iterations and then it remains there. The results further show that the P_ε evaluated using the WSRMax algorithm is highly dependent on ε. That is, the smaller the deviations in the $\hat{C}_{\text{subopt}}$ from the optimal C_{opt}, the larger the number of iterations required by the WSRMax algorithm to reach the expected target value P_ε. However, independent from the ε, proposed Algorithm 1 allows to find a suboptimal solution within a small number of iterations. Results further show that as the number of parallel initialization points are increased, P_ε become almost zero. This confirms that most of the time our proposed algorithm find the optimal solution.



(a) Average normalized weighted sum-rate deviation vs. Iterations for $K = 2$ users, $M = 8$ subcarriers and SNR = 10dB.

(b) Probability of missing global optimum vs. Iterations for $K = 2$ users, $M = 8$ subcarriers and SNR = 10dB.

Conclusions

A joint subcarrier and power allocation algorithm has been proposed for maximizing the weighted sum-rate in multiuser OFDMA downlink systems which can be used as a building block for more general resource allocation algorithms for *long term evolution* downlink transmissions. The proposed solution method is an simple extension of our previous work [3] and the algorithm is based on an approximated primal decomposition based method. The original nonconvex optimization problem is divided into two subproblems which can be solved independently. Numerical results show that, despite its reduced computational complexity, the proposed algorithm provides very close to optimal performance.

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