ANALYTICAL LINK DISTANCE DERIVATIONS FOR 3D CO-CHANNEL INTERFERENCE SCENARIOS

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INTRODUCTION

Co-channel interference is one of the fundamental limiting factors for the radio capacity of wireless systems. Femtocells [1] have gained a lot of interest lately as an effective solution to increase spectral efficiency. Femtocells need also to cope with interference (originating, e.g., from other femtocells and macrocells in the neighborhood). Paper [2] presented simulation based co-channel co-existence performance evaluation between indoor femtocells and a co-located macrocell. This contribution complements those studies with applicable analytical link distance models.

LINK DISTANCE DISTRIBUTIONS IN 3D INDOOR LAYOUT

Monte Carlo simulations are often used for averaging performance over, e.g., spatial geometry variations. Typically simulations require a high number of realizations, i.e., high complexity to be statistically accurate enough. From the computational point of view the closed-form analytical models are preferable whenever available. Here the link distance probability density functions (PDFs) are solved analytically for the case of having multiple square-shaped (femto)cells next to each other as shown in Fig. 1. Following the principle that link distance PDF is equal to the arc length normalized by the sectional area [3], and after some trigonometry and geometry exercises, the distance from AP23 in Fig. 1 to user planes in cells numbered by 0–5 can be solved and expressed as

$$p_0(d) = \begin{cases} \frac{2\pi d}{a^2}, & x_0 \le d \le x_1, \\ \frac{2\pi d}{a^2} - \frac{8d}{a^2}\cos^{-1}\left(\frac{a}{2\sqrt{d^2-h^2}}\right), & x_1 < d \le x_2, \end{cases} \tag{1}$$

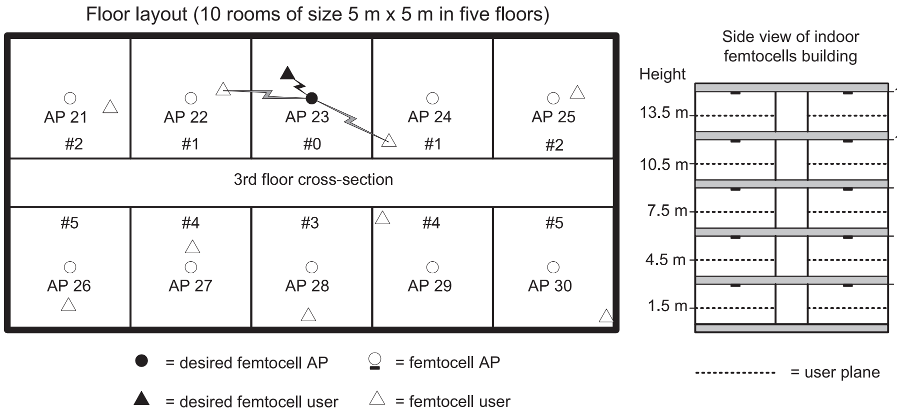

Figure 1: Floor layout for the femtocells.

$$p_1(d) = \begin{cases} \frac{2d}{a^2}\cos^{-1}\left(\frac{a}{2\sqrt{d^2-h^2}}\right), & x_1 \le d \le x_2, \\ \frac{2d}{a^2}\sin^{-1}\left(\frac{a}{2\sqrt{d^2-h^2}}\right), & x_2 < d \le x_5, \\ \frac{2d}{a^2}\left[\sin^{-1}\left(\frac{a}{2\sqrt{d^2-h^2}}\right) - \cos^{-1}\left(\frac{3a}{2\sqrt{d^2-h^2}}\right)\right], & x_5 < d \le x_6, \end{cases} \quad (2)$$

$$p_2(d) = \begin{cases} \frac{2d}{a^2}\cos^{-1}\left(\frac{3a}{2\sqrt{d^2-h^2}}\right), & x_5 \le d \le x_6, \\ \frac{2d}{a^2}\sin^{-1}\left(\frac{a}{2\sqrt{d^2-h^2}}\right), & x_6 < d \le x_{11}, \\ \frac{2d}{a^2}\left[\sin^{-1}\left(\frac{a}{2\sqrt{d^2-h^2}}\right) - \cos^{-1}\left(\frac{5a}{2\sqrt{d^2-h^2}}\right)\right], & x_{11} < d \le x_{12}, \end{cases} \quad (3)$$

$$p_3(d) = \begin{cases} \frac{2d}{a^2}\cos^{-1}\left(\frac{a/2+2}{\sqrt{d^2-h^2}}\right), & x_3 \le d \le x_4, \\ \frac{2d}{a^2}\sin^{-1}\left(\frac{a}{2\sqrt{d^2-h^2}}\right), & x_4 < d \le x_8, \\ \frac{2d}{a^2}\left[\sin^{-1}\left(\frac{a}{2\sqrt{d^2-h^2}}\right) - \cos^{-1}\left(\frac{3a/2+2}{\sqrt{d^2-h^2}}\right)\right], & x_8 < d \le x_9, \end{cases} \quad (4)$$

$$p_4(d) = \begin{cases} \frac{d}{a^2}\left[\cos^{-1}\left(\frac{a/2+2}{\sqrt{d^2-h^2}}\right) - \sin^{-1}\left(\frac{a}{2\sqrt{d^2-h^2}}\right)\right], & x_4 \le d \le x_7, \\ \frac{d}{a^2}\left[\sin^{-1}\left(\frac{3a}{2\sqrt{d^2-h^2}}\right) - \sin^{-1}\left(\frac{a}{2\sqrt{d^2-h^2}}\right)\right], & x_7 < d \le x_9, \\ \frac{d}{a^2}\left[\sin^{-1}\left(\frac{3a}{2\sqrt{d^2-h^2}}\right) - \cos^{-1}\left(\frac{3a/2+2}{\sqrt{d^2-h^2}}\right)\right], & x_9 < d \le x_{10}, \end{cases} \quad (5)$$

$$p_5(d) = \begin{cases} \frac{d}{a^2}\left[\cos^{-1}\left(\frac{a/2+2}{\sqrt{d^2-h^2}}\right) - \sin^{-1}\left(\frac{3a}{2\sqrt{d^2-h^2}}\right)\right], & x_7 \le d \le x_{10}, \\ \frac{d}{a^2}\left[\sin^{-1}\left(\frac{3a/2+2}{\sqrt{d^2-h^2}}\right) - \sin^{-1}\left(\frac{a/2+2}{\sqrt{d^2-h^2}}\right)\right], & x_{10} < d \le x_{13}, \\ \frac{d}{a^2}\left[\sin^{-1}\left(\frac{3a/2+2}{\sqrt{d^2-h^2}}\right) - \cos^{-1}\left(\frac{5a}{2\sqrt{d^2-h^2}}\right)\right], & x_{13} < d \le x_{14}, \end{cases} \quad (6)$$

where d denotes distance, a is the side length of the square cell, h is the height difference between link ends, and limits $x_i, i = 0, ..., 14$ are defined in Table 1.

Fig. 2 shows one floor link distance PDFs according to (1)–(6) with $h = 1.5$ m. Fig. 3 depicts the corresponding cumulative distribution functions (CDFs) that can be realized by integrating PDF over the whole distance range. For further numerical analyses or simulations it is now possible to select, e.g., worst, median or best case link distances as 0th, 50th or 100th percentiles of CDFs.

Finally, Fig. 4 gives an example how the analytical link distance models relate to the Monte Carlo simulation based approach in pure femtocell interference limited scenario. Averaged random snapshot simulation and 40th percentile link distance CDF provide close match whereas extremes illuminate the expected range of performance variations.

References

[1] V. Chandrasekhar, J. G. Andrews, and A. Gatherer, "Femtocell networks: a survey," *IEEE Commun. Mag.*, vol. 46, pp. 59–67, Sep. 2008.

[2] P. Pirinen, "Co-channel co-existence study of outdoor macrocell and indoor femtocell users," in *Proc. European Wireless 2010 Conference (EW'10)*, Lucca, Italy, Apr. 2010, pp. 207–213.

[3] S. W. Oh and K. H. Li, "Effect of circular-cell approximation on the forward-link BER performance of a power-controlled CDMA system," in *Proc. IEEE Global Telecommunications Conference (Globecom'99)*, Rio de Janeiro, Brazil, Dec. 1999, pp. 2472–2476.

Table 1: Distances in (1)–(6).

Distance	Value	Example: $(a = 5, h = 1.5)$
x_0	h	1.50
x_1	$\sqrt{\frac{a^2}{4} + h^2}$	2.92
x_2	$\sqrt{\frac{a^2}{2} + h^2}$	3.84
x_3	$\sqrt{\frac{a^2}{4} + 2a + 4 + h^2}$	4.74
x_4	$\sqrt{\frac{a^2}{2} + 2a + 4 + h^2}$	5.36
x_5	$\sqrt{\frac{9a^2}{4} + h^2}$	7.65
x_6	$\sqrt{\frac{5a^2}{2} + h^2}$	8.05
x_7	$\sqrt{\frac{5a^2}{2} + 2a + 4 + h^2}$	8.87
x_8	$\sqrt{\frac{9a^2}{4} + 6a + 4 + h^2}$	9.62
x_9	$\sqrt{\frac{5a^2}{2} + 6a + 4 + h^2}$	9.94
x_{10}	$\sqrt{\frac{9a^2}{2} + 6a + 4 + h^2}$	12.20
x_{11}	$\sqrt{\frac{25a^2}{4} + h^2}$	12.60
x_{12}	$\sqrt{\frac{13a^2}{2} + h^2}$	12.84
x_{13}	$\sqrt{\frac{13a^2}{2} + 2a + 4 + h^2}$	13.37
x_{14}	$\sqrt{\frac{17a^2}{2} + 6a + 4 + h^2}$	15.77

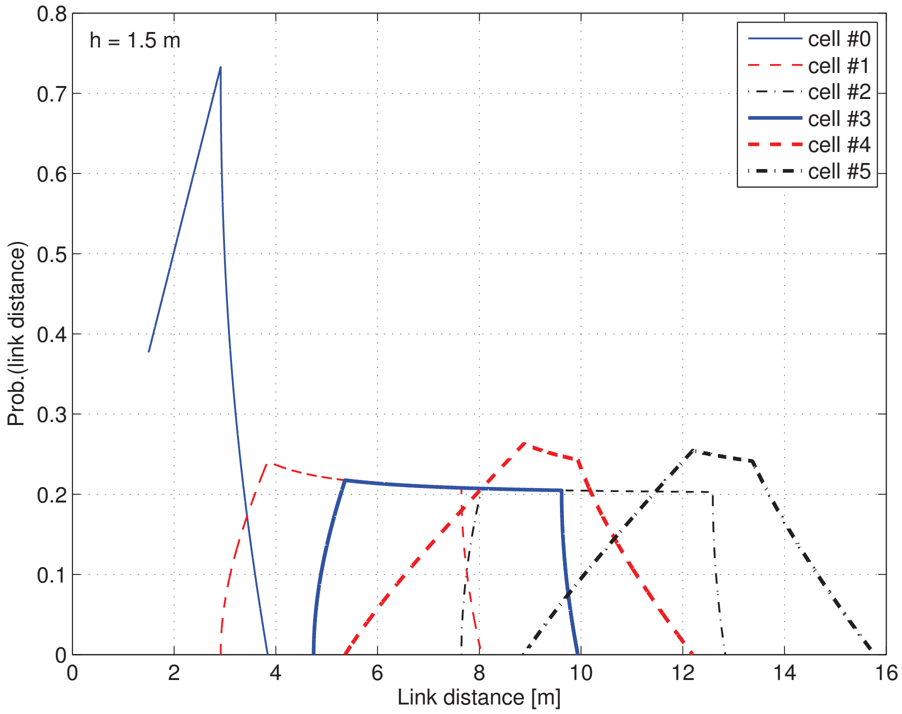

Figure 2: Probability density functions of interference link distances.

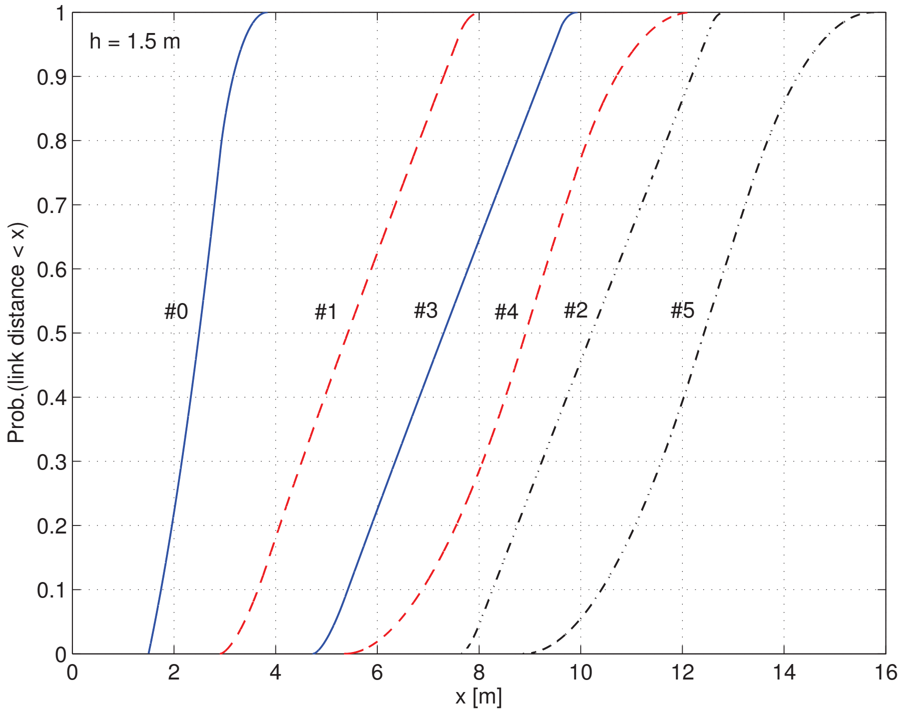

Figure 3: Cumulative distribution functions of interference link distances.

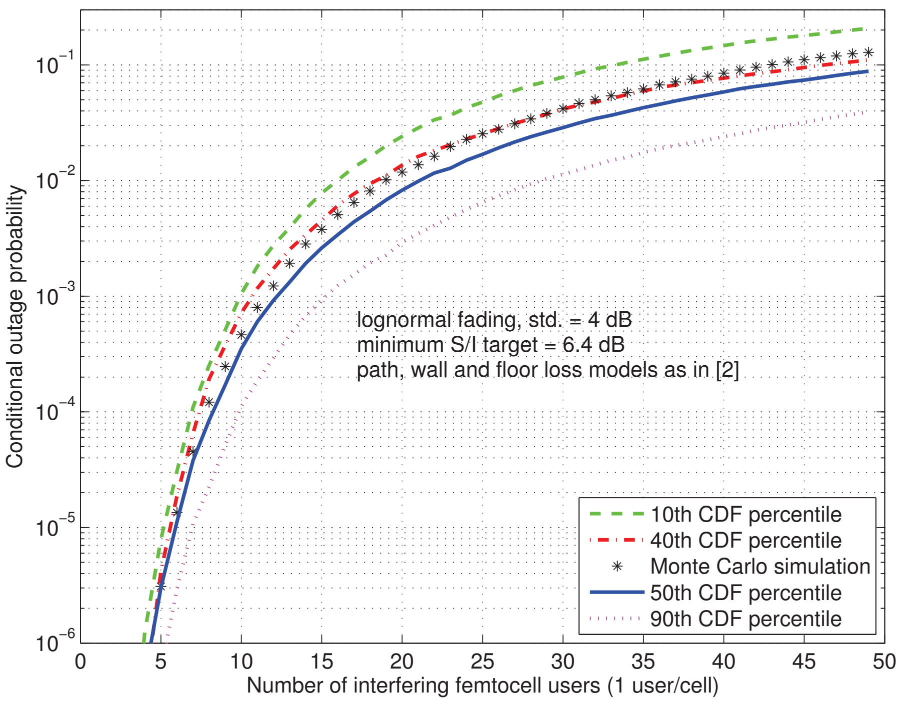

Figure 4: Femtocell interference limited conditional outage probability of the desired link.