Generalized Exponential Decay Model for Power–Delay Profiles of Multipath Channels

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INTRODUCTION

We study the modelling of generic multipath channels that, as illustrated in Figure 1, may comprise a line-of-sight path, specular and diffuse reflections as well as single and multiple scattering. We assume propagation environment in which the conditions of wide-sense stationary uncorrelated scattering (WSSUS) [1] are satisfied and channels remain approximately stationary during measuring or data frame transmission.

Our objective is to develop a generalized model for power–delay profiles (PDPs). Given impulse response $h(t)$ of a fading channel, the PDP represents the mean power of multipath components at delay t:

$$P(t) = \mathcal{E}\{|h(t)|^2\}. \tag{1}$$

The expectation operation $\mathcal{E}\{\cdot\}$ denotes statistically the average over the distribution of fast fading and the sample average in the context of measurements. Total transmit (Tx) and receive (Rx) powers are given by P_{Tx} and $P_{\text{Rx}} = \int_{-\infty}^{\infty} P(t)\ \mathrm{d}t$, respectively.

PDP modelling is essential for the characterization of propagation environments. It yields many well-known metrics such as the lag τ (the largest τ for which $P(t) = 0$ if $t < \tau$), the channel gain $g = P_{\text{Rx}}/P_{\text{Tx}}$, the mean delay $\mu = (1/P_{\text{Rx}}) \int_{-\infty}^{\infty} tP(t)\ \mathrm{d}t$, and the mean square delay spread $\sigma^2 = (1/P_{\text{Rx}}) \int_{-\infty}^{\infty} (t-\mu)^2 P(t)\ \mathrm{d}t$. The Fourier transform of the PDP gives the frequency correlation function (FCF) which is used for determining the channel coherence bandwidth. The PDP can be also used for evaluating the signal-to-interference ratio (SIR) of orthogonal frequency division multiplexing (OFDM) transmission.

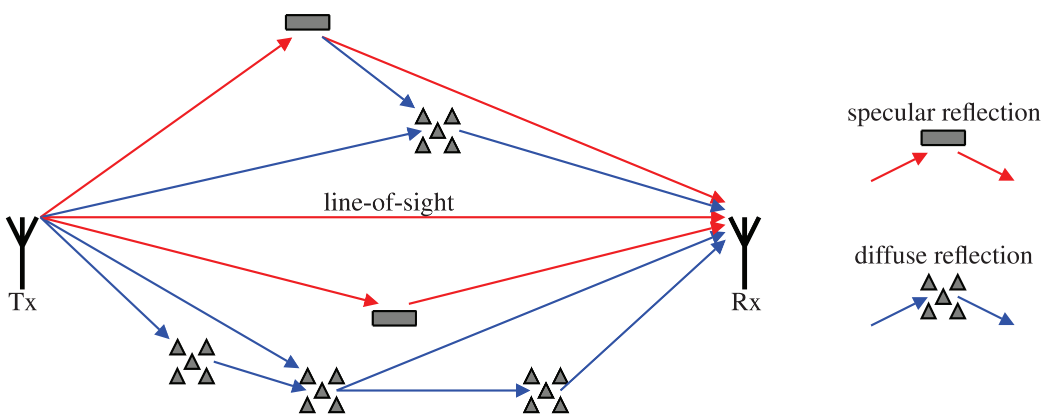


Figure 1: Conceptual multipath radio channel.

CLUSTERED POWER–DELAY PROFILE

We model the PDP as a composite of N clusters as shown in Figure 1. In theory $N \to \infty$ but in practice N should be quite small which imposes a tradeoff between modelling fidelity and analytical tractability. Each cluster comprises a large number of small-scale multipath components that propagate through the same set of large-scale specular and diffuse scatterers. We let K_n denote the order of the nth cluster, i.e., the number of *diffuse* reflections at the propagation path.

The clustered PDP can be expressed as

$$P(t) = P_{\text{specular}}(t) + P_{\text{diffuse}}(t) = \sum_{n=1}^{N} P_{K_n}(t, g_n, \tau_n, S_n) \tag{2}$$

in which $P_{K_n}(t, g_n, \tau_n, S_n)$ is the PDP of the nth cluster, characterized by gain g_n, lag τ_n, and the set of delay spread parameters $S_n = \{\sigma_{n,1}, \sigma_{n,2}, \ldots, \sigma_{n,K_n}\}$ associated with the K_n diffuse reflections. For simplicity, we consider the practical case in which $\sigma_{n,k}$, $k = 1, 2, \ldots, K_n$, are distinct for all n. The total channel gain and lag become $g = \sum_{n=1}^{N} g_n$ and $\tau = \min_n \tau_n$, respectively. Figure 2 shows an example of a clustered PDP.

Specular clusters

The specular part $P_{\text{specular}}(t)$ models the potential line-of-sight (LOS) path and unscattered (mirror-like) reflections. As $K = 0$ for a specular cluster, its PDP becomes simply

$$P_0(t, g, \tau, \{\}) = g P_{\text{Tx}}\, \delta(t - \tau) \tag{3}$$

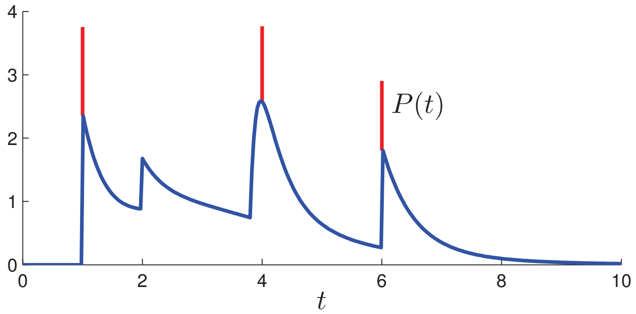
where $\delta(\cdot)$ denotes the unit impulse for which $\delta(t) = 0$ when $t \neq 0$ and $\int_{-\infty}^{\infty} \delta(t)\ \mathrm{d}t = 1$. The only parameters are gain g and lag τ, and the mean delay becomes $\mu = \tau$.

Diffuse clusters with single scattering

The diffuse part $P_{\text{diffuse}}(t)$ models the dense multipath components (DMC) resulting from the combination of single and multiple scattering. Single scattering ($K = 1$) is modelled with the classic exponential PDP [2]:

$$P_1(t, g, \tau, \{\sigma\}) = \frac{g P_{\text{Tx}}}{\sigma} e^{-\frac{t-\tau}{\sigma}} \mathrm{U}(t - \tau), \tag{4}$$

where $\mathrm{U}(\cdot)$ denotes the unit step for which $\mathrm{U}(t) = 0$ if $t < 0$ and $\mathrm{U}(t) = 1$ if $t \geq 0$. The parameters are gain g, lag τ and root mean square (RMS) delay spread σ, and the mean delay becomes $\mu = \sigma + \tau$. The classic exponential PDP may be used also for modelling specular clusters (with small σ) because $\lim_{\sigma \to 0} P_1(t, g, \tau, \{\sigma\}) = P_0(t, g, \tau, \{\})$.



n	K_n	g_n	τ_n	S_n
1	0	1.4	1.0	$\{\}$
2	0	1.2	4.0	$\{\}$
3	0	1.1	6.0	$\{\}$
4	1	1.2	1.0	$\{0.5\}$
5	1	0.4	2.0	$\{0.5\}$
6	1	0.8	6.0	$\{0.5\}$
7	2	1.2	3.8	$\{0.1, 0.4\}$
8	3	3.0	1.0	$\{0.9, 1.0, 1.1\}$

Figure 2: Example power–delay profile ($N = 8$, $P_{\text{Tx}} = 1$).

GENERALIZED EXPONENTIAL DECAY MODEL FOR MULTIPLE SCATTERING

We generalize now the classic exponential PDP given in (4) for modelling clusters with multiple scattering ($K > 1$) in $P_{\text{diffuse}}(t)$. The model is based upon the fact that a propagation path with $K_1 + K_2$ reflections can be partitioned into a cascade of two propagation paths with K_1 and K_2 reflections. This is equivalent to the convolution of partition PDPs:

$$P_{K_1+K_2}(t, g_1 g_2, \tau_1 + \tau_2, S_1 \cup S_2) = \frac{1}{P_{\text{Tx}}} P_{K_1}(t, g_1, \tau_1, S_1) * P_{K_2}(t, g_2, \tau_2, S_2) \tag{5}$$

for any $K_1 \geq 0$ and $K_2 \geq 0$.

Any cluster with K reflections can be partitioned into a cascade of K single-scattering clusters by applying (5) repeatedly. After calculating the multiple convolutions, we get

$$\begin{aligned} P_K(t, g, \tau, S) &= \frac{1}{P_{\text{Tx}}^{K-1}} P_1(t, g_1, \tau_1, \{\sigma_1\}) * P_1(t, g_2, \tau_2, \{\sigma_2\}) * \ldots * P_1(t, g_K, \tau_K, \{\sigma_K\}) \\ &= \sum_{k=1}^{K} \left(\prod_{l=1,\, l \neq k}^{K} \frac{\sigma_k}{\sigma_k - \sigma_l} \right) P_1(t, g, \tau, \{\sigma_k\}) \end{aligned} \tag{6}$$

in which $g = \prod_{k=1}^{K} g_k$ and $\tau = \sum_{k=1}^{K} \tau_k$ denote the total gain and lag of the cluster, respectively. Thus, a K-fold multiple-scattering cluster is effectively equal to the superposition of K single-scattering clusters interfering both constructively and destructively.

APPLICATION TO THE ANALYSIS OF OFDM TRANSMISSION

In OFDM transmission, orthogonal frequency-domain subcarriers are transformed into a time-domain signal with inverse fast Fourier transform (FFT) of length T_{FFT}. As a countermeasure against multipath fading, cyclic prefix (CP) of length T_{CP} is added to mitigate inter-symbol interference and to facilitate simple frequency-domain equalization. The receiver is synchronized to begin the symbol demodulation at time instant τ_{TOR} which is referred to as the time of reference (TOR).

One application for the PDP model in the analysis of OFDM systems is given by its FCF and the coherence bandwidth. They provide useful information for system design, e.g., when choosing subcarrier spacing and frequency-domain pilot structures.

Another application for the PDP model is in the evaluation of the signal-to-interference ratio:

$$\text{SIR} = \frac{P_{\text{U}}}{P_{\text{ICI}} + P_{\text{ISI}}} \tag{7}$$

for which received power $P_{\text{Rx}} = P_{\text{U}} + P_{\text{ICI}} + P_{\text{ISI}}$ is divided into useful signal power P_{U}, inter-carrier interference (ICI) power P_{ICI}, and inter-symbol interference (ISI) power P_{ISI}.

Multipath components that arrive outside of the CP window cause ICI and ISI and their effect becomes more pronounced with excessive delay spread. Thus, the power of multipath components can be divided into useful and interference power based on their delays w.r.t. the TOR. For this purpose, we apply a piecewise linear weighting function [3]

$$w(t) = \frac{1}{T_{\text{FFT}}} \max\left\{0, \min\left\{T_{\text{FFT}}, t - \tau_{\text{a}}, \tau_{\text{d}} - t\right\}\right\} \tag{8}$$

with breakpoints $\tau_{\text{a}} = \tau_{\text{TOR}} - T_{\text{FFT}}$, $\tau_{\text{b}} = \tau_{\text{TOR}}$, $\tau_{\text{c}} = \tau_{\text{TOR}} + T_{\text{CP}}$, and $\tau_{\text{d}} = \tau_{\text{TOR}} + T_{\text{CP}} + T_{\text{FFT}}$. The weighting approach is demonstrated in Figure 3.

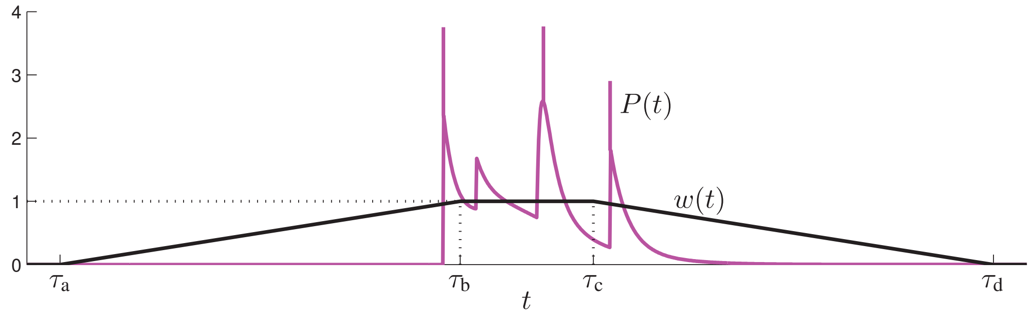


Figure 3: The weighting function.

As a variation of the discrete-case analysis presented in [3], the signal power components can be obtained from the continuous PDP by calculating the following integrals:

$$P_{\mathrm{U}} = \int_{-\infty}^{\infty} w^2(t)P(t)\ \mathrm{d}t, \tag{9}$$

$$P_{\mathrm{ICI}} = \int_{-\infty}^{\infty} w(t)P(t)\ \mathrm{d}t - P_{\mathrm{U}}, \tag{10}$$

$$P_{\mathrm{ISI}} = \int_{-\infty}^{\infty} \big(1 - w(t)\big)\ P(t)\ \mathrm{d}t. \tag{11}$$

Thus, the shape of the PDP defined by the parameters in (2) and the physical layer OFDM parameters (T_{FFT}, T_{CP}) are involved in a tradeoff determining the performance of the modulation scheme. Furthermore, the PDP model allows to compare different receiver time synchronization schemes through the TOR in the presence of ICI and ISI. In [4], we apply this methodology for the analysis of a full-duplex repeater link.

CONCLUSION

We proposed a generalized model for the power–delay profiles of multipath radio channels. The model is a composite of exponentially-decaying clusters that represent the line-of-sight path, specular reflections, and diffuse reflections with single and multiple scattering. The PDP is useful for the characterization of propagation environment and, in particular, it facilitates analytical evaluation of the performance of OFDM transmission.

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