A Glance on Simple Electromagnetic Homogenization Procedures

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INTRODUCTION

In electromagnetics, homogenization means replacing the complex heterogeneous microstructure of a material by a macroscopically homogeneous model. In other words, the effective parameters, electric permittivity ϵ_{eff} and magnetic permeability μ_{eff}, of the material are sought. Homogenization has a long history. However, the problem is still topical, as more complex structures can nowadays be more accurately modeled using computational methods [1, 2]. Using this knowledge, the material parameters can be tuned in a desired way by constructing artificial composite materials.

In this presentation we consider a geometrically simple example, a simple cubic lattice of dielectric spheres in vacuum (see Fig. 1). The spherical inclusions with relative permittivity $\epsilon_{\text{i}} = 10$ occupy a volume fraction $p = 1/10$ of the total volume. The relative permittivity of the external vacuum is denoted by $\epsilon_{\text{e}} = 1$. The spheres are assumed nonmagnetic and only the effective relative permittivity ϵ_{eff} is studied.

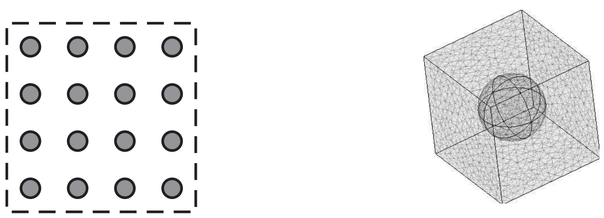

Figure 1: A simple cubic lattice (2D projection) (left) and one unit cell (right).

First, we consider the electrostatic case with an infinite lattice using classical mixing formulas. Then, we study a dynamic case where a propagating wave interacts with the composite material. The material parameters are retrieved numerically using the reflection and transmission data computed from a slab-like sample of the considered material.

The idea of homogenization is based on the condition that the intrinsic inhomogeneities of the material must be notably small compared with the wavelength of the external field. We want to see how quickly and strongly the obtained permittivity starts to deviate from the static reference value as the frequency and the electrical size of the unit cells of the lattice are increasing. We are also seeking limitations after which the obtained parameters are no longer reliable. Our current research deals with determining these limits more accurately and finding more stable methods for composite material homogenization [3, 4].

FROM STATICS TO DYNAMICS

In the literature, there exists a great variety of different mixing formulas [5]. A simple formula suitable for a cubic lattice of spheres is the Maxwell Garnett mixing rule, which using the given parameters ($p = 1/10$, $\epsilon_\mathrm{i} = 10$ and $\epsilon_\mathrm{e} = 1$) gives

$$\epsilon_\mathrm{MG} = \epsilon_\mathrm{e} + 3p\epsilon_\mathrm{e}\frac{\epsilon_\mathrm{i} - \epsilon_\mathrm{e}}{\epsilon_\mathrm{i} + 2\epsilon_\mathrm{e} - p(\epsilon_\mathrm{i} - \epsilon_\mathrm{e})} \approx 1.243. \quad (1)$$

A better estimate can be obtained using e.g. Lord Rayleigh mixing formula, but with a volume fraction this small, (1) can be considered a reference accurate enough.

The dynamic case is modeled using COMSOL MULTIPHYSICS, which is a commercial software based on the finite element method (FEM). We are applying the S-parameter retrieval, also know as Nicolson–Ross–Weir technique [6, 7]. The effective permittivity of the material is retrieved based on the simulated reflection and transmission data. In this case, we cannot handle an infinite amount of bulk material. Instead we have to study only a thin sample of the composite. There are two limitations for the sample slab. Naturally, the material parameters should not depend on the amount of the material. This means that the slab must be thick enough to include a sufficient number of unit cells to behave like a bulk material. On the other hand, the weakness of the NRW technique is that the electric thickness of the sample should not exceed $\lambda/2$, as then the solution becomes non-unique. Even though the correct branch closest to the physical solution could be chosen, the solution is contaminated by Fabry–Pérot type of resonances, which occur when the slab thickness becomes an integer multiple of one half wavelength.

Fig. 2 presents the obtained permittivitties for different slabs as a function of frequency. The frequency scale is normalized by f_{20} that denotes the frequency where the thickness of one unit cell is $\lambda/20$ compared with the quasistatic effective wavelength. The effect of frequency dispersion is clearly seen. A slab consisting of only one layer gives a higher permittivity compared with the thicker slabs, which indicates that it does not behave like a bulk material. By adding more layers, the permittivity in the static limits starts to converge towards the Maxwell Garnett approximation (1).

The most distinguishable features in Fig. 2 are, however, the unexpected resonances occurring for slabs with 5 and 9 layers within the shown frequency range. These are due to the aforementioned Fabry–Pérot phenomenon where the total thickness of the sample becomes $\lambda/2$ and the reflection from the slab vanishes. Nevertheless, this natural occurrence should not be seen as a resonance in the permittivity curve. Moreover, at the observed (anti-)resonances, the permittivity is decreasing with increasing frequency, which violates the principle of causality. Therefore, the retrieval technique is succesful only below the first Fabry–Pérot resonance, where also the size of one unit cell remains very small.

CONCLUSIONS

Homogenization of a simple dielectric cubic lattice composite was studied. The results of the dynamic S-parameter simulations were compared with an analytical static estimate. It was seen that with increasing frequency the retrieved permittivity starts to increase and notably deviate from the static value. When using the S-parameter retrieval, the amount of material, i.e., the number of unit cells must be sufficient for the sample to behave as a bulk material. On the other hand, the total thickness of the sample should remain below one half wavelegth, as otherwise the result is contaminated by Fabry-Pérot type of resonances. These restrictions together state that the size of one unit cell must remain extremely small.

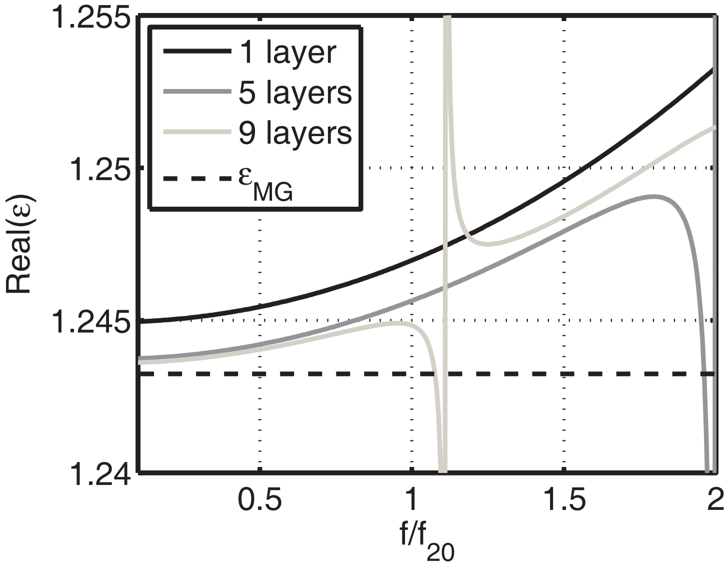

Figure 2: Retrieved permittivities for slabs with different number of layers as a function of (normalized) frequency.

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