Approximating ideal boundary conditions in electromagnetics with penetrable interfaces

Ari Sihvola, Henrik Wallén, Pasi Ylä-Oijala, Ismo V. Lindell

Aalto University School of Science and Technology
Department of Radio Science and Engineering
P.O. Box 13000, FI–00076 AALTO, Finland
Email: firstname.lastname@tkk.fi

In problems of electromagnetics, accepted field solutions need to satisfy the Maxwell equations. In addition, for the fields to be unique in a given domain of interest, they also need to obey boundary conditions on the surface of this domain. A well-known boundary condition is the so-called perfect electric conductor (PEC) boundary condition which requires that the tangential (parallel to the surface) electric field vanishes at this boundary. By duality, the PMC (perfect magnetic conductor) boundary condition means that the tangential magnetic field becomes zero at the boundary. A generalization of these two conditions is the PEMC (perfect electromagnetic conductor) boundary condition [1] which means that a linear combination of the electric and magnetic fields cannot have a tangential component. Recently, also boundary conditions involving *normal* components of the fields have been introduced [2, 3]. For example, the so-called DB boundary condition forces the normal components of the electric and magnetic flux densities to vanish.

Ideal boundary conditions are useful in solving electromagnetic problems because the space behind this boundary does not have any effect on the fields. In real-world problems, however, we encounter interfaces, not absolute boundaries. In other words, the fields penetrate through material interfaces, and the structure behind the interface needs to be taken into account in the exact solution of the problem. However, if the material contrast over the interface is very strong, the replacement of the interface by an ideal boundary condition can be a very reasonable approximation. For example, in microwave engineering applications, the surface of a very good conductor can approximate quite well the PEC boundary.

In addition to approximating a material interface by a boundary condition, which can be called the *analytic approach* to electromagnetic problems, also the reverse approximation can be taken, the *synthetic approach*. By this latter approach we mean that the boundary condition is the primary object of our study, and we ask the question which type of material structure would simulate the effect on electromagnetic fields as closely as possible (when compared with the ideal boundary). Intuitively, one expects that the material parameters of such a synthetic structure would need to be very different in magnitude compared with the corresponding parameters in the domain where the fields are calculated, in other words, the medium on the other side of the interface should possess *extreme material parameters*, to borrow a term from recent metamaterials literature [4].

As an example, Figure 1 displays a comparison of the effect of a boundary condition and material interface in the case of plane wave scattering by a sphere. Shown is the scattering efficiency (total scattering cross section normalized by the geometrical cross section) of the sphere which has a size parameter of the value 4. The dotted line shows the value of a DB sphere, $Q_{sca} = 2.542$, and the two curves show the behavior of the cross section for material spheres when their parameters vary. The permittivity and permeability are equal in both cases and approach zero inversely with the parameter *b*. For the solid curve, the parameters are real and for the dashed curve they are purely imaginary. Both penetrable spheres clearly approach the DB sphere in terms of this scattering quantity. Indeed, the relative difference is already down to 1% when the sphere permittivity and permeability are 0.1 ($b = 10$).

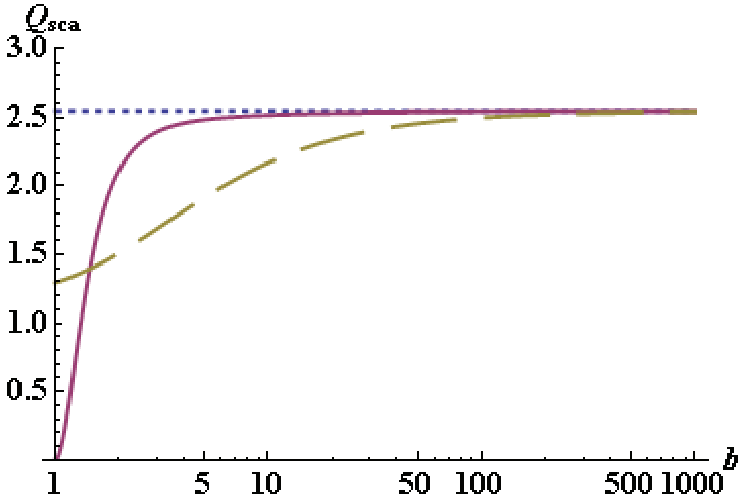

Figure 1: The scattering efficiency of a sphere with size parameter (wave number times radius) of 4. Dotted blue line: DB sphere, solid red line: homogeneous sphere with equal (real) permittivity and permeability of value *1/b*, dashed green line: homogeneous sphere with equal (imaginary) permittivity and permeability of value j/*b*.

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