Statistical analysis of incoherent scatter radar data

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INTRODUCTION

The ionosphere is a partly ionised region of the atmosphere, extending from about 50 km to above 1000 km altitude. Weak scattering of radio waves from thermal fluctuations in the ionospheric plasma is called incoherent scatter. Incoherent scatter radars are high power large aperture radar systems, which detect properties of the incoherent scatter – usually its autocorrelation function – in order to gain information of prevailing ionospheric plasma parameters, such as electron density, electron temperature, ion temperature and line-of-sight plasma velocity. The measurement consists of two parts: autocorrelation function of the scattering process is first measured at different heights, and plasma parameters are then estimated from the measured autocorrelation functions.

The ionosphere consists of regions called "D", "E", and "F". The D region is the lowest one extending from 50 to 90 km, the E region is located around 100 km, and the area of strongest day time ionisation around 300 km altitude is the F region. As the plasma parameters in these regions are quite different, also the correlation time of the scattering process has large variations with height: the F region correlation time is short enough to make that part of the radar target severely overspread, whereas the D region is clearly an underspread target. The different regions thus require considerably different measurement techniques, making combined measurements of all ionospheric regions difficult.

Incoherent scatter radars use special modulation methods, which allow the autocorrelation function of the overspread target to be measured with a range resolution much better than transmission pulse length. Most commonly the desired resolution is achieved by combining data from several pulses in a special decoding process. The decoding works well only if the modulation fulfils certain, rather strict, requirements. These requirements can be almost completely relaxed if the decoding process is replaced with methods based on statistical inversion. Using this possibility, we have been able to design so-called multi purpose modulations, which are suitable for simultaneous autocorrelation function measurements from all regions of the ionosphere.

The fit of plasma parameters is a non-linear inverse problem, which is usually solved by means of iterative techniques. The analysis searches for a minimum of a cost function, and the first located minimum is accepted, though there is no guarantee that it is a global one. Errorbars of the plasma parameters are then usually derived from linearised theory, assuming Gaussian error distributions. We have applied a Monte Carlo technique to the plasma parameter fit, which allows the whole error distribution of the plasma parameters to be investigated, without restrictive prior assumptions about its shape. The method can be used to investigate the validity of the iteration results.

LAG PROFILE INVERSION

Due to the typically short correlation time of the incoherent scatter signal, conventional pulse compression by means of phase coding and decoding cannot be used as such with incoherent scatter radars. However, it is possible to first calculate lagged products of the received signal, and then decode these so-called lag profiles. The decoder output then consists of samples of the autocorrelation function at different heights, and the full autocorrelation functions can be constructed by simply re-organising and averaging the decoder output samples. The widely used alternating code technique exploits the lag profile decoding: the modulation consists of a set of phase coded pulses, which produce a complementary code set at all time lags shorter than the pulse length. The alternating code technique thus enables perfect side lobe cancellation in autocorrelation function measurement of a severely overspread target.

The echo signal received with a radar is convolution of the transmission modulation and the target. In simple pulse compression such modulations are used, whose autocorrelation function is exactly or nearly zero at all non-zero time lags. The received signal can then be decoded by correlating it with the modulation. This approach is valid also in lag profile decoding, if the target is replaced by the true lag profile, and the transmission modulation is replaced by its lagged product, the range ambiguity function. If other methods than decoding are available for the deconvolution, one gains the freedom to use arbitrary modulation patterns, as long as the modulation is known to the data analyst. This possibility is exploited in lag profile inversion method [1], where a specialised software package for large linear inverse problems, FLIPS [2], is used for solving the deconvolution problem. The method is identical to decoding if alternating codes are used, but complete side lobe cancellation is possible for any other modulation as well. The price of using non-perfect modulations is paid in the form of increased variance of the results.

The input data of lag profile inversion is raw voltage samples of both attenuated transmitted signals and the received signals. Accurate information of the transmission waveform is thus always available, and we have gained the freedom to transmit literally anything – within the limitations set by the radar hardware.

MULTI PURPOSE MODULATIONS

The difficulty of combined measurements of the whole ionosphere arises from the fact, that the D-region would require pulses to be transmitted with short inter-pulse periods, whereas long pulses transmitted with long inter-pulse periods are sufficient for the upper parts of the ionosphere. A natural choice would be to transmit pulses with non-uniform inter-pulse periods but, because decoding requires pulses to be transmitted with uniform inter-pulse periods, this has not been possible.

As the lag profile inversion analysis [1] has provided us the freedom to transmit any modulation allowed by the radar transmitter, we have been able to perform the first multi purpose experiments with the EISCAT radars. The analysis of combinations of different pulse lengths and inter-pulse periods was first demonstrated [4] and a pulse coding method known as aperiodic transmitter coding [5] was then applied to incoherent scatter radar measurements with some modifications [3]. In addition to the ionospheric works, the aperiodic modulations have been used in special space debris measurements [6], where also plasma parameters were derived from the same data.

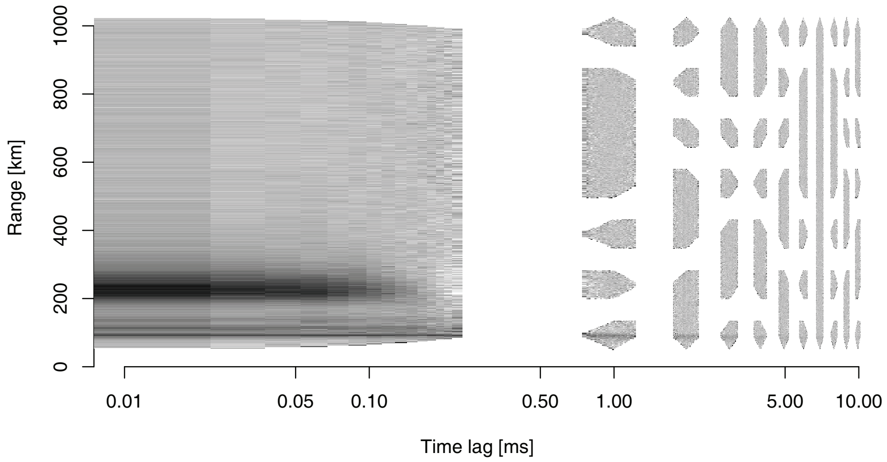

Figure 1: Real part of incoherent scatter autocorrelation function, measured with the PPATC technique [3] in Tromsø, Norway, November 28 2008, 12:32 UT.

MONTE CARLO INVERSION OF PLASMA PARAMETERS

The iterative fit of plasma parameters can be validated by means of Monte Carlo methods, which provide the full error distribution of the plasma parameters. We have recently implemented a variant of the random walk Metropolis algorithm [7], the adaptive Metropolis [8], to the plasma parameter fit. The adaptive sampling simplifies the analysis from the user point-of-view, as the algorithm automatically tunes itself according to the target distribution. Efficient sampling can be achieved with very little prior knowledge of the error distributions. Examples of electron density and ion temperature distributions are given in Fig. 2.

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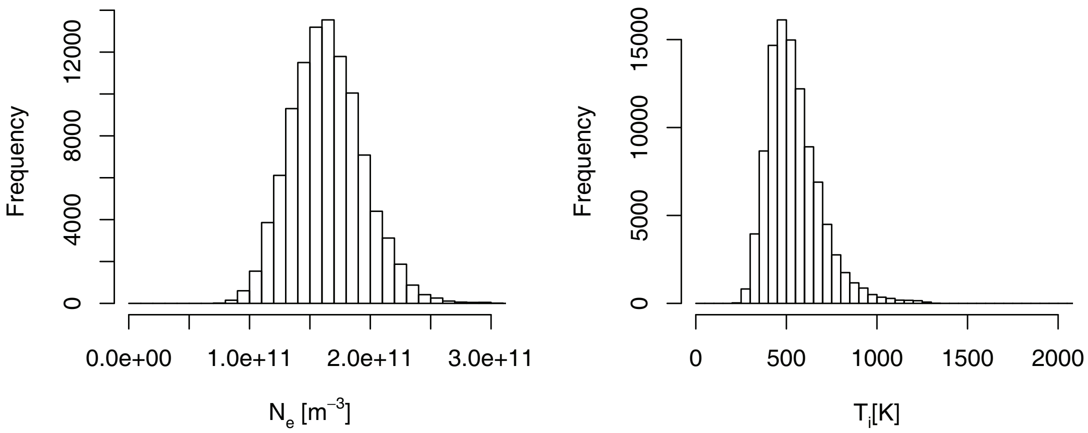

Figure 2: Marginal distributions of electron density (left) and ion temperature (right) at 130 km altitude above Tromsø, Norway, November 27 2008, 20:22 UT. Adaptive Metropolis algorithm was used to sample 100 000 points. The histograms were made using the last 90 000 points, whereas the first 10 000 were excluded as a burnin period.

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