Processing of radiation patterns using spherical harmonic expansions

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INTRODUCTION

Measured or simulated radiation patterns of antennas are typically represented on a regular grid in the spherical coordinate system. When such data is rotated, the density of the far-field data in the new coordinate system varies greatly. For example, if the new z axis is pointing to the old x axis direction, there are much fewer measurement points near the new north pole than there was for the north pole of the original data, which makes standard Cartesian interpolation difficult.

In this paper we examine the use of spherical harmonic expansions to represent the far-field antenna radiation patterns and thereby giving easy tools for rotating and interpolating the data. The results can also be used in sensor array signal processing algorithms.

SPHERICAL HARMONIC EXPANSIONS

In the near and far fields it is possible to represent the fields caused by the antennas using the spherical wave expansions, i.e. as a sum of vector spherical wave functions [3]. In these expansions, the number of terms must be controlled carefully in order to catch all the energy in the fields but to avoid any numerical problems caused by the use of too many terms [3].

In this paper we represent the far field using scalar spherical harmonic expansions with vector coefficients as introduced in [2]. The authors only discussed the representation of far-field patterns from simulations but the methodology can easily be applied to measured radiation pattern data as well.

The spherical harmonics are defined by

$$Y_l^m(\theta,\phi) = b_{lm} P_l^m(\cos\theta) e^{im\phi},$$

where b_{lm} are normalization coefficients, $P_l^m(\cos\theta)$ are the associated Legendre functions of the first kind [1], and l is the degree and m the order of the expansion.

The expansion of the far field in terms of vector coefficients $\mathbf{a}_{lm}$ is given by

$$\mathbf{E}(\hat{\mathbf{r}}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \mathbf{a}_{lm} Y_l^m(\hat{\mathbf{r}}) \tag{1}$$

where $\hat{\mathbf{r}}$ is the unit vector to the direction represented by θ and ϕ.

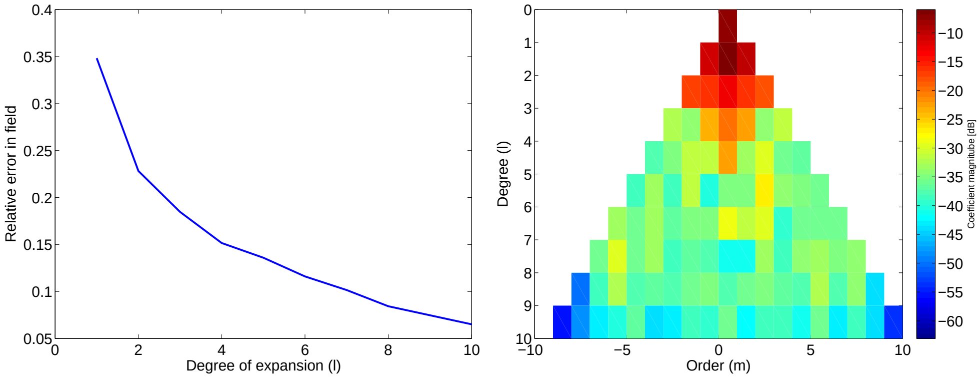

Figure 1: Left: convergence of the spherical harmonic expansion. Relative error is defined as $\|\mathbf{E}_m - \mathbf{E}_r\|/\|\mathbf{E}_m\|$ where the subscripts refer to measured and reconstructed fields. Right: magnitudes of the coefficients.

Due to the orthogonality of the spherical harmonics, integration of the far field with the spherical harmonics would give the vector coefficients directly. However, to cope with incomplete measurement data we obtained the coefficients from a least squares solution of (1) for each of the Cartesian components separately.

Fig. 1 shows the convergence of the expansion for a measured radiation pattern of an antenna array and the values of the expansion coefficients. In the comparison, the r component in the expansion has been neglected. The plots show that most of the energy is concentrated in the low degree modes and adding more modes reduces the reconstruction error. For measurement data there is an optimal number of levels yielding the best representation of the field if the dominating error is measurement noise.

ROTATION AND INTERPOLATION

It is very simple to rotate a radiation pattern expressed using the spherical harmonics. Instead of rotating the pattern itself, consider the pattern as fixed and rotate the new coordinate system with respect to the pattern. Then transform the grid of the new coordinate system to the original coordinate system and just read the value of the spherical harmonic expansion. Interpolation also comes naturally in this procedure.

Many measurement systems are not able to measure the whole spherical surface. With the current method the incomplete measurement data can be easily fitted to the spherical harmonic expansion and then extrapolated beyond the original measurement area. Furthermore, it is not required to measure the radiation pattern in an equidistant angular grid.

References

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