EIGENVALUE FRACTION DISTRIBUTION IN I.I.D. AND CORRELATED FLAT RAYLEIGH FADING CHANNELS

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INTRODUCTION

In this report we study the post processing signal-to-interference (SINR) difference distributions of a precoded MIMO with a maximum of two spatially multiplexed streams in both i.i.d. and correlated flat Rayleigh fading channels. When adaptive modulation and coding (AMC) is used the transmitter needs to be informed about the channel conditions in order to adjust the transmission rate. A channel quality indicator (CQI) for both data streams is fed back from the receiver to inform the transmitter about the channel conditions. To reduce the amount of feedback, the CQI of one stream may be considered relative to the CQI of the other stream. The distributions of SINR difference may be used for optimization of the relative CQI offset levels.

The precoding considered is based on singular value decomposition (SVD), which means that the channel is orthogonalized [1]. It follows that after zero forcing (ZF) receiver, the distribution of the post processing SINR difference between the streams corresponds to the probability density function (PDF) of the fraction of the eigenvalues of the channel in logarithmic domain. The PDFs of the eigenvalue fraction can be obtained in closed form for both i.i.d. and correlated flat Rayleigh fading channels from the joint eigenvalue distributions.

EIGENVALUE FRACTION DISTRIBUTIONS

With the parametrization of [1] after a ZF receiver the post processing SINR values can be written as $SINR_1^{(ZF)} = \frac{\gamma}{(r\lambda_1^{-1}+(1-r)\lambda_2^{-1})}$ and $SINR_2^{(ZF)} = \frac{\gamma}{(r\lambda_2^{-1}+(1-r)\lambda_1^{-1})}$, where r is an orthogonalization parameter. When $r=1$ or $r=0$ and we have $SINR_k^{(ZF)} = \gamma\lambda_k$.

For the flat Rayleigh fading channel, $\mathbf{HH}^{\mathrm{H}}$ follows a Wishart-type distribution. The PDF of the ordered eigenvalues of i.i.d. Rayleigh fading channel can be found in [2]. For 2×2 MIMO the joint distribution of λ_1 and λ_2 reads

$$p(\lambda_1,\lambda_2) = \frac{1}{2}(\lambda_1-\lambda_2)^2 \exp\left(-\lambda_1-\lambda_2\right) \tag{1}$$

The PDF of $u=\frac{\lambda_1}{\lambda_2}$ can be integrated from that and reads

$$p(u) = \frac{6(u-1)^2}{(u+1)^4} \tag{2}$$

For the correlated scenario the distribution can be obtained from the formulas e.g. in [3]. We assume correlation only at the transmitter since the receive correlation is typically small. For 2×2 the PDF of u in closed from reads

$$p(u) = \frac{\kappa_1^2 \kappa_2^2 (u-1)^2 ((\kappa_1^2 + \kappa_2^2)(1+u+u^2) + \kappa_1 \kappa_2 (1+4u+u^2))}{(\kappa_1^2 u + \kappa_2^2 u + \kappa_1 \kappa_2 (1+u^2))^3} \tag{3}$$

Fig. 1 and 2 show the analytical eigenvalue fraction PDFs compared to the simulated SINR difference distributions in i.i.d. and correlated channels, respectively.

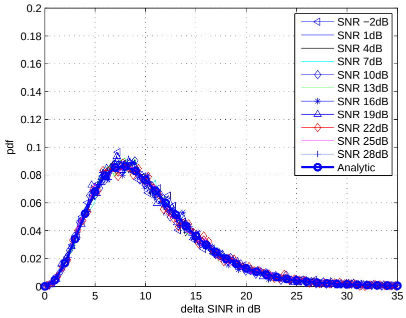

Figure 1: Eigenvalue fraction distribution compared to simulated SINR difference distributions in i.i.d. channel.

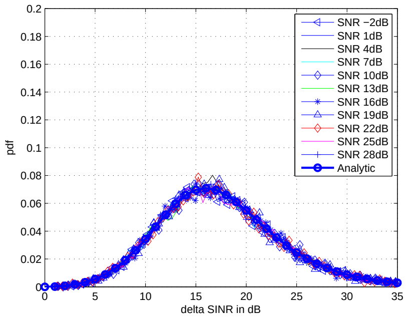

Figure 2: Eigenvalue fraction distribution compared to simulated SINR difference distributions in ULA correlated channel with correlation parameter 0.9.

References

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