ESSENTIAL EQUATIONS IN CIRCUIT ANALYSIS

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INTRODUCTION

Usually the special program is needed for circuit analysis. This report includes essential information concerning ladder circuits. They can be used in spreadsheet or more effective mathematical programs.

IMPEDANCES, CHAIN MATRIXES AND SCATTERING PARAMETERS

Lumped components are resistors, inductors and capacitors. They form impedances and admittances. The whole circuit is usually characterized with scattering parameters. Chain matrixes are needed as intermediate results. Following figure and equations show the basic circuits, immittances and chain matrixes.

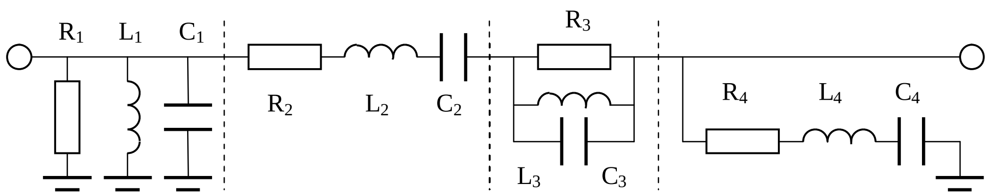


Figure 1. Basic lumped circuits.

$$Y_1 = \frac{1}{R_1} + \frac{1}{j\omega L_1} + j\omega C_1 \quad (1)$$

$$Z_2 = R_2 + j\omega L_2 + \frac{1}{j\omega C_2} \quad (2)$$

$$Y_3 = \frac{1}{R_3} + \frac{1}{j\omega L_3} + j\omega C_3 \quad (3)$$

$$Z_4 = R_4 + j\omega L_4 + \frac{1}{j\omega C_4} \quad (4)$$

$$A_{1234} = \begin{bmatrix} 1 & 0 \\ Y_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1/Y_3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/Z_4 & 1 \end{bmatrix} \quad (5)$$

Important transmission line circuits are stubs, which can be shorted, open, capacitor ended and capacitor coupled. The equations include hyperbolic functions. The arguments are propagation coefficient and line length.

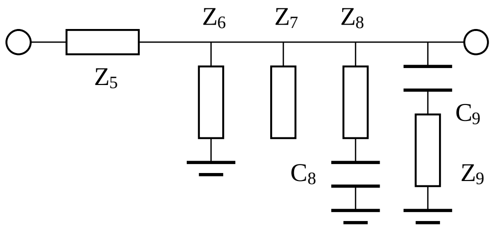


Figure 2. Basic transmission line circuits.

$$A_5 = \begin{bmatrix} \cosh(\gamma_5 l_5) & \sinh(\gamma_5 l_5) Z_5 \\ \sinh(\gamma_5 l_5)/Z_5 & \cosh(\gamma_5 l_5) \end{bmatrix} \quad (6)$$

$$Y_6 = \coth(\gamma_6 l_6)/Z_6 \quad (7)$$

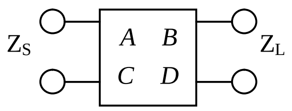
$$Y_7 = \tanh(\gamma_7 l_7)/Z_7 \quad (8)$$

$$Y_8 = \frac{j\omega C_8 + \tanh(\gamma_8 l_8)/Z_8}{1 + j\omega C_8 Z_8 \tanh(\gamma_8 l_8)} \quad (9)$$

$$1/Y_9 = \frac{1}{j\omega C_9} + \tanh(\gamma_9 l_9) Z_9 \quad (10)$$

$$A_{6789} = \begin{bmatrix} 1 & 0 \\ Y_6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_7 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_9 & 1 \end{bmatrix} \quad (11)$$

The scattering coefficients can be calculated from the chain matrix. Source and load impedances have to be defined.



$$s_{11} = \frac{AZ_L + B - CZ_S Z_L - DZ_S}{AZ_L + B + CZ_S Z_L + DZ_S} \quad (12)$$

$$s_{12} = \frac{2(AD - BC)\sqrt{Z_S Z_L}}{AZ_L + B + CZ_S Z_L + DZ_S} \quad (13)$$

$$s_{21} = \frac{2\sqrt{Z_S Z_L}}{AZ_L + B + CZ_S Z_L + DZ_S} \quad (14)$$

$$s_{22} = \frac{-AZ_L + B - CZ_S Z_L + DZ_S}{AZ_L + B + CZ_S Z_L + DZ_S} \quad (15)$$