PEMC material in electromagnetics

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The concept of **P**erfect **E**lectro**M**agnetic **C**onductor (PEMC) medium was introduced in [1], and has been since analyzed in the literature by the group at the TKK and elsewhere in the world. It is not an overstatement to say that PEMC is a very fundamental type of material, at the same time extremely simple and very complex. In light of the present wave of interest in *metamaterials* [2], the properties of PEMC medium deserve a detailed study.

To define PEMC, let us consider the following constitutive relations between the electric ($\mathbf{E}$) and magnetic ($\mathbf{H}$) fields and electric ($\mathbf{D}$) and magnetic ($\mathbf{B}$) displacements:

$$\mathbf{D} = M\mathbf{B}, \quad \mathbf{H} = -M\mathbf{E} \tag{1}$$

Here M is a real scalar admittance-type quantity.

Such a material response type is encountered most naturally when considering electromagnetic equations in terms of differential forms in four-dimensional Minkowskian representation. In fact, in this case—given by Equation (1)—the medium condition between the electromagnetic two-forms [3]

$$\Psi = \mathbf{D} - \mathbf{H} \wedge \mathbf{d}\tau \tag{2}$$

$$\Phi = \mathbf{B} + \mathbf{E} \wedge \mathbf{d}\tau \tag{3}$$

in the simple form

$$\Psi = M\Phi$$

Concerning the notation, $\wedge$ is the wedge (outer) product and $\mathbf{d}\tau$ the time-like one-form.

Here the two-forms each combine two three-dimensional quantities as the space-like and time-like components. The extreme simplicity of this relation ($\Psi = M\Phi$ with M a single pseudoscalar) motivates us to call the medium (1) as *the one and only isotropic medium*. Such a medium is invariant in any affine transformations, including the Lorentz transformation. This means that the medium appears the same for any observer traveling with constant velocity. In four-dimensional electrodynamics, this medium goes also under the name *axion* medium [4].

As is shown in [1, 5], this medium is a generalization of perfect electric and magnetic conductors (PEC, PMC). The label PEMC is therefore a natural one to describe it. In every spatial point within PEMC, a linear combination of fields vanishes ($\mathbf{H} + M\mathbf{E} = 0$) as also a combination of displacements is zero ($\mathbf{D} - M\mathbf{B} = 0$), which is seen from Equation (1). The special case $1/M = 0$ returns us PEC: ($\mathbf{E} = 0, \mathbf{B} = 0$) and we arrive at PMC when $M = 0$: ($\mathbf{H} = 0, \mathbf{D} = 0$).

A perhaps more common way than Equation (1) to represent the material relations is the one when the displacements are given as responses to the fields:

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \epsilon & \xi \\ \zeta & \mu \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \mathsf{C} \cdot \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \tag{4}$$

where the material matrix C contains the four scalars $\epsilon, \xi, \zeta, \mu$. Equation (4) can be written as

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{H} \end{pmatrix} = \frac{1}{\mu} \begin{pmatrix} \epsilon\mu - \xi\zeta & \xi \\ -\zeta & 1 \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{B} \end{pmatrix} \tag{5}$$

Comparing this relation (5) with Equation (1), we arrive at the PEMC constitutive matrix

$$\mathsf{C} = q \begin{pmatrix} M & 1 \\ 1 & 1/M \end{pmatrix}, \quad \text{with} \quad |q| \to \infty \tag{6}$$

In this poster, electromagnetic problems involving PEMC boundaries are reviewed which may find interesting applications in microwave engineering. For a more detailed description about the analysis and applications of the PEMC medium, see [6].

References

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