Electromagnetic DB boundary

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INTRODUCTION

A novel set of boundary conditions, requiring vanishing of the normal components of the $\mathbf{D}$ and $\mathbf{B}$ vectors at the boundary surface, is suggested. Labeled as DB boundary its basic properties are studied in this paper. Reflection of any plane wave with complex propagation factor from the planar DB boundary is analyzed. It is shown that waves polarized TE and TM with respect to the normal of the boundary are reflected as from PEC and PMC planes, respectively and the same is valid for any field outside the sources. Realization of such a boundary is discussed.

BOUNDARY CONDITIONS

Electromagnetic problems bounded by some closed surface S require boundary conditions at S for the fields to have unique solutions. Let us assume S is the plane $z = 0$ plus the infinite half sphere around the half space $z > 0$. For time-harmonic fields with sources at the finite distance from the origin the boundary conditions at the infinite half sphere are the well-known radiation conditions requiring vanishing of field as $1/r$. At the plane $z = 0$ linear conditions would require two scalar equations for the electromagnetic field vectors. Let us list a few such conditions. Here the subscript $_t$ denotes component transverse to the normal unit vector $\mathbf{u}_z$.

- The PEC condition $\mathbf{E}_t = 0$ or $\mathbf{u}_x \cdot \mathbf{E} = 0$ and $\mathbf{u}_y \cdot \mathbf{E} = 0$
- The PMC condition $\mathbf{H}_t = 0$ or $\mathbf{u}_x \cdot \mathbf{E} = 0$ and $\mathbf{u}_y \cdot \mathbf{E} = 0$
- The PEMC condition $(\mathbf{H} + M\mathbf{E})_t = 0$
- The soft-and-hard condition $\mathbf{u}_x \cdot \mathbf{E} = 0$ and $\mathbf{u}_x \cdot \mathbf{H} = 0$
- The impedance condition $(\mathbf{E} + \overline{\overline{\mathsf{Z}}} \cdot \mathbf{H})_t = 0$ for some surface impedance dyadic $\overline{\overline{\mathsf{Z}}}_s$

All of the previous conditions were associated to the electromagnetic field vectors $\mathbf{E}$ and $\mathbf{H}$. Let us now form another set of boundary conditions for the vectors $\mathbf{D}$ and $\mathbf{B}$ by defining

$$\mathbf{u}_z \cdot \mathbf{D} = 0, \quad \mathbf{u}_z \cdot \mathbf{B} = 0. \tag{1}$$

Let us call such a boundary by the name DB boundary for brevity.

The corresponding conditions for the $\mathbf{E}$ and $\mathbf{H}$ vectors depends on the medium in the half space $z > 0$. Assuming simple isotropic medium with permittivity ϵ and permeability μ, (1) is equivalent with the conditions

$$\mathbf{u}_z \cdot \mathbf{E} = 0, \quad \mathbf{u}_z \cdot \mathbf{H} = 0. \tag{2}$$

The same is also valid for general bi-isotropic media.

REFLECTION FROM IDEAL DB BOUNDARY

To see how a plane wave is reflected from a boundary satisfying (1), let us first assume that the incident and reflected plane-wave fields have the form

$$\mathbf{E}^i(\mathbf{r}) = \mathbf{E}^i e^{-j\mathbf{k}^i\cdot\mathbf{r}}, \quad \mathbf{H}^i(\mathbf{r}) = \mathbf{H}^i e^{-j\mathbf{k}^i\cdot\mathbf{r}}, \tag{3}$$

$$\mathbf{E}^r(\mathbf{r}) = \mathbf{E}^r e^{-j\mathbf{k}^r\cdot\mathbf{r}}, \quad \mathbf{H}^r(\mathbf{r}) = \mathbf{H}^r e^{-j\mathbf{k}^r\cdot\mathbf{r}}, \tag{4}$$

where the wave vectors are $\mathbf{k}^i = -k_z\mathbf{u}_z + \mathbf{K}$ and $\mathbf{k}^r = k_z\mathbf{u}_z + \mathbf{K}$. Here $\mathbf{K}$ is a vector transverse to $\mathbf{u}_z$ and may have complex components.

Assuming now that the boundary conditions are valid for the total fields $\mathbf{E}^i + \mathbf{E}^r$ and $\mathbf{H}^i + \mathbf{H}^r$ in the form (2), applying $\mathbf{k}^i\cdot\mathbf{E}^i = 0$, $\mathbf{k}^r\cdot\mathbf{E}^r = 0$ and the Maxwell equations we can express the boundary conditions in the form

$$\mathbf{K}\cdot(\mathbf{E}^i - \mathbf{E}^r) = 0. \tag{5}$$

$$(\mathbf{u}_z\times\mathbf{K})\cdot(\mathbf{E}^i + \mathbf{E}^r) = 0. \tag{6}$$

From these we obtain the following relation between the reflected and incident field components transverse to the z axis:

$$\mathbf{E}^r_t = \overline{\overline{\mathsf{R}}}_t\cdot\mathbf{E}^i_t, \tag{7}$$

with the reflection dyadic defined by

$$\overline{\overline{\mathsf{R}}}_t = \frac{1}{K^2}(\mathbf{K}\mathbf{K} - (\mathbf{u}_z\times\mathbf{K})(\mathbf{u}_z\times\mathbf{K})). \tag{8}$$

Obviously, eigenvectors of the reflection dyadic are $\mathbf{K}$ and $\mathbf{u}_z\times\mathbf{K}$ corresponding to the respective eigenvalues $+1$ and -1. Actually, the former defines a wave TM polarized, and the latter TE polarized, with respect to the z axis. This means that the DB boundary appears as a PMC boundary for the TM wave and a PEC boundary for the TE wave. Since these are valid for any plane waves, they are valid for any linear combinations of plane waves and, ultimately, for any fields.

REALIZATION OF DB BOUNDARY

A realization of the DB boundary appears possible in terms of an interface of a uniaxially anisotropic medium whose permittivity and permeability dyadics are defined by

$$\overline{\overline{\epsilon}} = \epsilon_z\mathbf{u}_z\mathbf{u}_z + \epsilon_t\overline{\overline{\mathsf{I}}}_t, \quad \overline{\overline{\mu}} = \mu_z\mathbf{u}_z\mathbf{u}_z + \mu_t\overline{\overline{\mathsf{I}}}_t, \tag{9}$$

where $\overline{\overline{\mathsf{I}}}_t$ is the transverse unit dyadic. In fact, requiring that the axial parameters vanish, $\epsilon_z\to 0$, $\mu_z\to 0$, the z components of the $\mathbf{D}$ and $\mathbf{B}$ fields vanish in the medium and, because of the continuity across the interface, the conditions (1) are valid at the interface. A medium (9) with vanishing axial parameters is a challenge for metamaterials research.

For small but finite values of ϵ_z and μ_z the above characteristics for the TE and TM waves fail in a small cone around the axial direction. Actually, if $\sqrt{\mu_t/\epsilon_t}$ equals the wave impedance of the medium in front of the interface, the TEM wave incident normally is not reflected at all. Thus, a slab of such a medium acts as a spatial filter for fields incident to the interface, reflecting all but those entering close to the normal direction.